proved (i) to be necessary over any E_m-field by consideration of the structure of the full linear permutation group. Extending this method, all Galois groups of x^m-a over R, possible under the conditions of the theorem, are established. (Received May 1, 1947.)

312. G. E. Wall: *Notes on binomic equations over an E_m-field.* II.

Let x^m-a be irreducible over the E_m-field R, and reducible over $R(\epsilon)$ (ϵ=primitive mth root of unity), and let $\alpha^m=a$. The form of the coefficient a is determined so that $R(\alpha, \epsilon)$ has a given Galois group $\mathfrak{G}$ over R (out of a class of admissible groups $\mathfrak{G}$ established in part I of this paper). Firstly, using the criteria of G. Darbi for normal binomic equations over an E_m-field (Annali di Matematica pura ed applicata (4) vol. 4 (1926)), the form of a is determined so that $\mathfrak{G}$ is of given order. These results are then applied to certain binomic subfields of $R(\alpha, \epsilon)$ in order to distinguish between groups $\mathfrak{G}$ of equal order. (Received May 1, 1947.)

ANALYSIS

313. R. P. Boas: *Some complete sets of analytic functions.*

In the following theorems are generalized results of Ibragimov (Bull. Acad. Sci. URSS. Sér. Math. vol. 11 (1947) pp. 75–100). Let $f(z)$ be analytic in $|y|<\pi$ and of period 2π. Let η and ζ be positive numbers with $\eta<\pi$, $\zeta\leqq\pi-\eta$. Let a set of lower density ζ/π of the Fourier coefficients of positive index of $f(z)$ not vanish. Let $\alpha_n=\beta_n+i\gamma_n$ be a sequence of complex numbers, $0\leqq\beta_n<2\pi$. Then $\{f(z+\alpha_n)\}$ is complete in $|z|<\zeta$ in the following cases. (1) The set $\{\gamma_n\}$ has a limit point in $|y|<\eta$. (2) The function $f(z)$ is entire, of order ρ and type σ_1, and $2\sigma_1<\rho$ lim inf $n|\gamma_n|^{1-\rho}$. (Received July 22, 1947.)

314. R. P. Boas and K. Chandrasekharan: *Derivatives of infinite order.*

Let $f(x)$ have derivatives of all orders in (a, b). The following theorems are proved. (1) If $f^{(n)}(x)\to g(x)$ for each x in (a, b), where $g(x)$ is finite, then $g(x)=Ae^x$. (2) If $f(x)$ belongs to a Denjoy-Carleman quasi-analytic class in the open interval (a, b), and $\lim_{n\to\infty} f^{(n)}(x_0)=L$ exists for a single x_0 in (a, b), then $f^{(n)}(x)\to Le^{x-x_0}$ in (a, b). These theorems answer in the affirmative questions raised by V. Ganapathy Iyer (J. Indian Math. Soc. N.S. vol. 8 (1944) pp. 94–108) and remain true if $f^{(n)}(x)\to g(x)$ in a more general sense. If $\{\lambda_n\}$ is a given sequence of constants, the following theorems are proved. (3) Let (*) $\lim_{n\to\infty} f^{(n)}(x)/\lambda_n=g(x)$, $a\leqq x\leqq b$. If lim inf $|\lambda_{n-1}/\lambda_n|=0$ and (*) holds uniformly, $g(x)\equiv 0$ in (a, b). If lim inf $|\lambda_{n-1}/\lambda_n|>0$ and (*) holds dominatedly, $g(x)=Ae^{Bx}$. (4) If lim sup $n^{-1}|\lambda_n|^{1/n}<\infty$ and (*) is true for each x in $a<x<b$, then $g(x)=Ae^{Bx}$. (Received June 26, 1947.)

315. V. F. Cowling: *Some results for factorial series.*

Let $f(z)=\sum_{n=0}^{\infty}a_n n!/z(z+1)\cdots(z+n+1)$ with abscissa of convergence $\tau<\infty$. Let $l-1<h<l$ where l is integral and positive but otherwise arbitrary. Denote by D the domain in the w-plane $\psi_2\leqq\text{Arg }(w-h)\leqq\psi_1$ where $0<\psi_1\leqq\pi/2$ and $-\pi/2\leqq\psi_2<0$. Let $a(w)$ be a function regular in D, with the possible exception of the point at infinity, for which $a(n)=a_n$, $n=0, 1, \cdots$. Suppose for $w=h+Re^{i\psi}$ in D and $R\geqq R_1$ that $|a(h+Re^{i\psi})|\leqq R^k \exp(-LR\sin\psi)$ where k is arbitrary and $0<L<2\pi$. Then

$f(z)$ may be continued analytically to any point of the plane except the points $z=0, -1, -2, \cdots$. (Received July 28, 1947.)

316. Evelyn Frank: *Continued fraction expansions for meromorphic functions.*

The continued fraction expansion $k_0\gamma_0+k_0(1-\gamma_0\bar{\gamma}_0)z/\bar{\gamma}_0z-1/k_1\gamma_1+k_1(1-\gamma_1\bar{\gamma}_1)z/\bar{\gamma}_1z-1/k_2\gamma_2+\cdots$, where the k_p are constants chosen so that $|\gamma_p|\neq 1$, converges uniformly over every bounded closed domain within $|z|<|r_1|<1$ to a meromorphic function $f(z)$ with poles at the points $r_1, r_2, \cdots, r_m$, where $0<|r_1|\leqq|r_2|\leqq\cdots\leqq|r_m|$. Conversely, every such function may be represented in that domain by a continued fraction of this form. The expansion is finite if $f(z)$ is a rational function of a particular form (Frank, Trans. Amer. Math. Soc. vol. 62 (1947)). Furthermore, the expansion may be continued analytically over the entire finite plane with the exception of the poles of $f(z)$, and these continuations converge uniformly to the values of the function in their respective circular domains which contain none of the poles of $f(z)$ and are of radius less than 1. This extends the theory of Schur (J. Reine Angew. Math. vol. 147 (1917)) for analytic functions with moduli less than 1 in the unit circle. Other forms of this continued fraction are found which connect with the preceding theory on such expansions, and certain non-circular convergence regions are shown. (Received July 28, 1947.)

317. K. O. Friedrichs: *Nonlinear hyperbolic differential equations for functions of two independent variables.*

For N functions u^n of x and y, systems of differential equations of the form $\sum_{n=1}^{N}(a^{mn}u_x^n+b^{mn}u_y^n)=g^m$, $m=1, \cdots, N$, are considered, in which the coefficients a, b, g depend on x, y, u. The values of u on a section of the initial line $y=0$ are prescribed such that the system is hyperbolic for them. In a neighborhood of this section, the unique existence of the solution with continuous second derivatives is proved if a, b, g and the initial value of u have continuous second derivatives. If these quantities have higher continuous derivatives, the same is shown to be true for the solution. The result applies to one equation of higher order for one function (an earlier paper of H. Lewy and the author on this subject contains an essential error). The proof consists in establishing sufficiently strong theorems for linear equations so that iterations can be carried out. The method of characteristics is used only for linear equations. (Received July 22, 1947.)

318. Philip Hartman: *Unrestricted solution fields of almost-separable differential equations.*

Let $g(x)$ and $f(t)$ be real continuous functions of a single variable for $-\infty<x<\infty$ and $0\leqq t<\infty$, respectively. Let $x=x(t)$ be a solution, on some t-interval $0\leqq t<\epsilon$, of the differential equation $dx/dt=g(x)+f(t)$. The question as to whether or not the solution $x=x(t)$ can be extended over the entire non-negative t-axis is considered; in the former case, the asymptotic behavior of $x(t)$, as $t\to\infty$, is also treated. The results obtained for these problems are then applied in a discussion of the asymptotic behavior of $y(t)$ and dy/dt, where $y=y(t)$ is a solution of the second order linear differential equation $d^2y/dt^2-(1+f(t))y=0$ and $f(t)$ is "small" for large values of t. (Received July 21, 1947.)

319. Einar Hille: *Non-oscillation theorems.*

Let $F(x)$ be defined for $x>0$, integrable over every finite interval, real-valued and of constant sign for large x. A necessary and sufficient condition that the equation $y''+F(x)y=0$ have a solution $y(x)$ with $y(x)\to 1$ when $x\to\infty$ is that $xF(x)\in L(1, \infty)$. Let $F(x)>0$ when $x>0$, form $g(x)=x\int_x^\infty (t)dt$, and let g_* and g^* be the inferior and superior limits of $g(x)$ when $x\to\infty$. A necessary condition that the differential equation have non-oscillatory solutions is that $g_*\leqq 1/4$, $g^*\leqq 1$, and a sufficient condition is that $g^*<1/4$. These inequalities are sharp. A logarithmic scale of non-oscillatory equations is determined. A new comparison theorem for a class of singular Riccati integral equations is one of the main tools in the investigation. (Received July 29, 1947.)

320. W. H. Ingram: *A generalized Pollard-Moore-Stieltjes integral.*

Let σ_k'' be any subdivision of $[ab]$ which includes among its points of division all points of discontinuity of $f(x)$ of the first kind and of a permissible second kind namely those for which $-\infty<f^U(x-)\equiv\lim\sup f(x-)=\lim\inf f(x-)=\lim\inf f(x+)<+\infty$, $\lim\sup f(x+)-\lim\inf f(x+)<+\infty$, at which points, for both kinds, the oscillation exceeds $\omega(k)$ and other points as required such that norm $\sigma_k''<\omega(k)$ norm σ_0''. The limit of the value of the interval function $\sum_{i/\sigma_k''}f^U(x_i+)\{g(x_{i+1})-g(x_i)\}$ as $k\to\infty$, $\omega(k)\to 0$, σ_k'' a refinement of σ_{k-1}'', x_i the ith point of σ_k'', $g(x)$ a function of bounded variation, is found to exist. The integral with variable upper limit $\int_a^x fdg$ is a function of bounded variation and, when $dy=y(x+)-y(x)$ unless zero in which case when $dy=(y')^+dx$, $d\int_a^x fdg=f(x+)dg(x)$. Also $\left|\int_a^b fdg-\sum_{i/\sigma}f^U(x_i+)\Delta_i g(x)\right|\leqq\Omega_{f/\sigma}v_g(b)$ where $\Omega_{f/\sigma}$ is the maximim of the oscillations of $f(x)$ in the (open) meshes of any arbitrary subdivision σ of $[ab]$ and v_g is the total variation of g. (Received June 16, 1947.)

321. W. H. Ingram: *The Green's matrix for a 1-dimensional boundary value problem.*

Let $Y(x)$ satisfy $dY=dH(x)Y$, det $Y\neq 0$, $H(x)$ of bounded variation. Then Y and Y^{-1} are of bounded variation and the solution of the problem $dY=dH(x)Y+df(x)I$, $LY(a)+RY(b)=0$, in which $df(x, x_0, \delta)$ is a scalar function zero everywhere except over the range $x_0\leqq x<x_0+\delta$ where it takes the value dx/δ, is, to within an ϵ, $\Gamma(x, x_0, \delta(\epsilon))\equiv 2^{-1}Y(x)\{s(x, x_0, \delta)+(LY(a)+RY(b))^{-1}(LY(a)-RY(b))\}Y^{-1}(\bar{x}_0)\bar{x}_0$ such that $\int_a^{x>x_0}Y^{-1}df=Y^{-1}(\bar{x}_0)\int_a^{x>x_0}df$, $a\leqq x_0<\bar{x}_0\leqq x_0+\delta\leqq b$; this mean value theorem is either true or becomes true as $\delta\to 0$; $s(x, x_0, \delta)=2\int_a^x df-1$. The Green's matrix is defined to be the limiting form of $\Gamma(x, x_0, \delta)$ as $\delta\to 0$ and satisfies the characterizing equations (i) $G(\xi+, \xi)-G(\xi, \xi)=I$, (ii) $d_xG(x, \xi)=dH(x)G(x, \xi)+2^{-1}d_x\operatorname{sign}(x-\xi-)I$, (iii) $Lg(a, \xi)+RG(b, \xi)=0$, $a<\xi<b$. The integral of the preceding abstract has application to the problem $dY=dH(x)Y+dF(x)$, $LY(a)+RY(b)=0$, $F(x)$ of bounded variation, and to the problem of Bull. Amer. Math. Soc. Abstract 51-11-219, for example, $Y(x)=\int_a^b G(x, \xi)dF(\xi)$, and is required in integrations involving $d_\xi G(x, \xi)$ which has a discontinuity, as a function of x, of the admissible second kind at $x=\xi$. (Received June 16, 1947.)

322. J. P. LaSalle: *Singular measurable sets and linear functionals.*

A class of measurable sets E is shown to be nonsingular if and only if given E and $0<\lambda<1$ there exists a measurable set $E'\subset E$ with the property that $\mu(E')=\lambda\mu(E)$, where μ is the measure. If one makes use of the above and known theorems for linear topological spaces, a theorem on the existence of a multiplicative-additive continuous functional on the space L^∞ is obtained, where the integral is over an abstract space and the measure of the space is not assumed to be finite. The result is an extension of a theorem given by Arens (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 931–935) and is similar to a theorem given by Day (Bull. Amer. Math. Soc. vol. 46 (1940) pp. 816–823). The method of proof gives a simple and direct proof of Day's theorem on the existence of linear functionals in the space L^p, $0<p<1$. (Received June 25, 1947.)

323. Morris Marden: *A note on lacunary polynomials.*

In this paper, it is proved that polynomials of the form $f(z)=a_0+a_1z+\cdots+a_pz^p+a_{n_1}z^{n_1}+\cdots+a_{n_k}z^{n_k}$, $a_0a_pa_n\cdots a_{n_k}\neq 0$, $1\leqq p<n_1<\cdots<n_k$, have at least p zeros in or on each of the circles $|z|\leqq Ar$ and $|z|\leqq Br$ where r is the radius of the circle $|z|=r$ containing all the zeros of the polynomial $F(z)=n_1n_2\cdots n_ka_0+(n_1-1)(n_2-1)\cdots(n_k-1)a_1z+\cdots+(n_1-p)(n_2-p)\cdots(n_k-p)a_pz^p$ and where $A=\csc^k(\pi/2p)$ and $B=\prod_{i=1}^{k}\prod_{j=1}^{p-1}(n_i+j)/(n_i-j)$. These bounds are derived by elementary methods based upon repeated use of the theorem that, if an nth degree polynomial has p zeros in $|z|\leqq 1$, its derivative has at least $p-1$ zeros in $|z|\leqq\csc\pi/2(n-p+1)=M$ as well as in $|z|\leqq\prod_{j=1}^{n-p}(n+j)/(n-j)=N$. The bound M is due to M. Marden (Trans. Amer. Math. Soc. vol. 45 (1939) pp. 335–368) and bound N is due to M. Bernacki (Bull. Soc. Math. France (2) vol. 69 (1945) pp. 197–203). (Received July 16, 1947.)

324. S. Minakshisundaram: *Notes on Fourier expansions.* III.

Let $F(e)$ be an additive function of bounded variation defined on Borel sets in the interval $(0\leqq x_1, x_2, \cdots, x_k\leqq 2\pi)$ and let $dF(e)\sim\sum C_{n_1\cdots n_k}e^{i(n_1x_1+\cdots+n_kx_k)}$, $C_{n_1\cdots n_k}=(1/(2\pi)^k)\int_0^{2\pi}\cdots\int_0^{2\pi}e^{-i(n_1x_1+\cdots+n_kx_k)}dF(e)$. Then (1) the series on the right called the *multiple Fourier-Stieltjes series* is almost everywhere summable $(\nu^2, (k-1)/2+\epsilon)$ for every $\epsilon>0$, that is, $\lim_{R\to\infty}\sum_{\nu^2\leqq R^2}(1-\nu^2/R^2)^{(k-1)/2+\epsilon}c_{n_1\cdots n_k}e^{i(n_1x_1+\cdots+n_kx_k)}$, $\nu^2=n_1^2+\cdots+n_k^2$ exists almost everywhere, and (2) at a point where the symmetric derivative of $F(e)$ exists the multiple Fourier-Stieltjes series is summable $(\nu^2, (k+1)/2+\epsilon)$. If $k=1$, this is equivalent to a familiar result in simple Fourier-Stieltjes series. (Received July 25, 1947.)

325. Josephine M. Mitchell: *Summability theorems for double orthogonal series whose coefficients satisfy certain conditions.*

Let (I) $\sum_{m,n=1}^{\infty}a_{mn}\phi_{mn}(x, y)$ be the orthogonal development of a function $f(x, y)\in L^2$ on the rectangle $Q(a\leqq x\leqq b, c\leqq y\leqq d)$, with respect to the complete orthonormal system $\{\phi_{mn}(x, y)\}$ $(m, n=1, 2, \cdots)$ $(\phi_{mn}(x, y)\in L^2)$. Let $s(m, n)$ be the mnth partial sum and $\sigma(m, n)$ the corresponding $(C, 1, 1)$ partial sum of (I). If the series (II) $\sum_{m,n=1}^{\infty}[\log(m+1)]^{1+\epsilon}a_{mn}^2$ and $\sum_{m,n=1}^{\infty}[\log(n+1)]^{1+\epsilon}a_{mn}^2$ (ϵ any arbitrary positive number) both converge, then series (I) is $(C, 1, 1)$ summable to s almost everywhere in Q, if and only if $\lim_{m,n\to\infty}s(2^m, 2^n)=s$ almost everywhere in Q. Under the weaker hypothesis that $\sum_{m,n=1}^{\infty}a_{mn}^2$ is convergent there exist two increasing sequences $\{m_i\}$ and $\{n_i\}$ of positive integers such that $\lim_{i\to\infty}\sigma(m_i, n_i)=s$ almost everywhere in Q, if and only if $\lim_{i\to\infty}s(2^{m_i}, 2^{n_i})=s$ almost everywhere in Q. If series (II) converges, then the $(C, 1, 1)$ summability of series (I) to s implies that $(1/mn)\sum_{\mu,\nu=1}^{m,n}|s_{\mu\nu}-s|$ and $(1/mn)\sum_{\mu,\nu=1}^{m,n}(s_{\mu\nu}-s)^2$ both approach 0 almost everywhere in Q as m and $n\to\infty$. Two properties of the Lebesgue functions, $\int_Q|\sum_{\mu,\nu=1}^{m,n}\phi_{\mu\nu}(x, y)\phi_{\mu\nu}(s, t)|dsdt$, are proved. The methods of proof used in this paper are generalizations of the one-variable proofs. Reference is made to a paper by R. P. Agnew (Proc. London Math. Soc. vol. 33 (1932)). (Received July 28, 1947.)

326. L. A. Ringenberg: *On the theory of the Burkill integral.*

The author is concerned with the regular integrals and derivatives of a function which is defined on a class of plane intervals satisfying a parameter of regularity condition. The results of several papers by Kempisty, Saks, Burkill, and others are coordinated; several converses and generalizations are given. In addition to results stated in terms of monotoneity, continuity, absolute continuity, bounded variation, and so on, several results are given for functions of type A (type of the Lebesgue area) and of type I. The type I property, suggested by Rado, is a simple characterization of a function of intervals which is the Lebesgue integral of its own derivative. (Received April 28, 1947.)

327. M. M. Schiffer: *Faber polynomials in the theory of univalent functions.*

Let D be a domain in the z-plane containing the point at infinity. Let $\xi=f(z)=z+b_0+b_1z^{-1}+\cdots$ be univalent in D and map it upon the domain Δ. The nth Faber polynomial $F_n(t)$ with respect to $f(z)$ is of nth degree in t and satisfies by definition $F_n[f(z)]=z^n+\sum_{m=1}^{\infty}\xi_{nm}z^{-m}$. If the domain Δ undergoes a variation $\xi^*=\xi+a\rho^2(\xi-\xi_0)^{-1}+b\rho^3(\xi-\xi_0)^{-2}+\cdots+\xi^*(\xi)$ univalent in Δ, $\rho>0$, $\xi_0\not\subset\Delta$ (cf. Schiffer, Proc. London Math. Soc. (2) vol. 44 (1938) pp. 432–449) the coefficients ξ_{nm} vary as follows: $\xi^*_{nm}=\xi_{nm}+a\rho^2m^{-1}F_n'(\xi_0)F_m'(\xi_0)+o(\rho^2)$. If $x_{\nu 1}$ $(\nu=1,\cdots,l)$ is an arbitrary set of complex numbers, the quadric $Q=\sum_{n,m=1}^{l}m\xi_{nm}x_nx_m$ is a functional of $f(z)$ (or Δ) with the variation formula $Q^*=Q+a\rho^2(\sum_{n=1}^{l}x_nF_n'(\xi_0))^2+o(\rho^2)$. Functionals with a variation formula of this type, that is, containing a perfect square, lead to extremum problems which are reducible to easily integrable differential equations. Grunsky's inequalities (Math. Zeit. vol. 45 (1939) pp. 29–61) and generalizations are obtained as particular applications. (Received July 14, 1947.)

328. I. J. Schoenberg: *Some analytical aspects of the problem of smoothing.*

The present note contains two parts dealing with two disconnected problems concerning linear transformations of the convolution type (1) $y_n'=\sum_{\nu=-\infty}^{\infty}L_{n-\nu}y_\nu$ $(n=\cdots,-2,-1,0,1,2,\cdots)$. First, the author solves the problem of finding the limiting form of the coefficients of the high order iterates of the transformation (1) which is also assumed to be symmetric, that is, $L_{-\nu}=L_\nu$. This problem was proposed by E. L. De Forest, an early writer on smoothing. Its present solution also helps to determine when a formula (1) may legitimately be called a smoothing formula. Second, the author studies formulas (1) which are variation-diminishing, that is, which have the property that the number of variations of signs in the sequence $\{y_n\}$ is never exceeded by the similar number in the sequence $\{y_n'\}$. Assuming that $L_n\to 0$ exponentially, as $n\to\pm\infty$, it is shown that (1) is variation-diminishing if and only if all minors (of all orders) of the matrix $\|L_{i-j}\|$ $(i,j=0,1,2,\cdots)$ are non-negative. A method of constructing such transformations in terms of their generating functions $F(z)=\sum_{-\infty}^{\infty}L_nz^n$ is also given. The converse proposition, to the effect that this construction furnishes all variation-diminishing transformations, is stated as a conjecture. (Received July 16, 1947.)

329. Walter Strodt: *The resolution into partial fractions of the reciprocal of an exponential polynomial.* Preliminary report.

Let $f(z) = \sum_{j=1}^{n} A_j \exp(\omega_j z)$, with the ω_j a set of complex numbers whose closed convex contains zero. Let $H(z) = 1/f(z)$. Let $Z_1, Z_2, \cdots$ be the distinct zeros of $f(z)$. Let $P_m(z)$ be the principal part of $H(z)$ at Z_m. Let $H(B)$ be finite. Then for every positive ϵ there exists a sequence of disjoint sets $\mathfrak{S}_1, \mathfrak{S}_2, \cdots$, whose union is the set $Z_1, Z_2, \cdots$, each $\mathfrak{S}_k$ containing at most $n-1$ points, and the diameter of each $\mathfrak{S}_k$ being less than ϵ, such that if $Q_k(z)$ is the sum over $\mathfrak{S}_k$ of the functions $P_m(z) - P_m(B)$, then $H(z) = H(B) + \sum_{k=1}^{\infty} Q_k(z)$ whenever $H(z)$ is finite. Moreover, for every pair of positive numbers δ_1, δ_2 the sequence $\mathfrak{S}_k$ can be chosen so that $Q_k(z) = (2\pi i)^{-1} \cdot \int_{C_k} H(t)(B-z)(t-z)^{-1}(t-B)^{-1}dt$, whenever $|z - Z_m| > \delta_1$ $(m = 1, 2, \cdots)$, where the C_k are contours of length less than δ_2 on which $H(t)$ is bounded, uniformly in k. This follows from well known theorems, due to C. E. Wilder, Tamarkin, and Pólya, about the zeros of $f(z)$. (Cf. Langer, *On the zeros of exponential sums and integrals*, Bull. Amer. Math. Soc. vol. 37 (1931).) (Received July 29, 1947.)

330. W. J. Thron: *Some properties of continued fractions* $1 + d_0 z + K(z/(1 + d_n z))$.

In this paper it is shown that for every power series (1) $1 + \sum_{n=1}^{\infty} c_n z^n$ there exists one and only one continued fraction (2) $1 + d_0 z + K(z/(1 + d_n z))$ which corresponds to the given power series (that is, the power series expansion of the nth approximant of (2) agrees with (1) up to and including the term $c_n z^n$). Further, conditions on the sequence $\{d_n\}$ which insure uniform convergence of (2) for: (a) $|z| < d$, (b) $|z| > M$, (c) $\beta_1 < \arg z < \beta_2$ are determined. Finally, various necessary conditions for the uniform convergence of (2) in a neighborhood of the origin are derived (for example, $\{d_n/n\}$ must be bounded). (Received June 18, 1947.)

331. J. L. Walsh: *The critical points of linear combinations of harmonic functions.*

Let R be a region bounded by a Jordan curve C, let the function $U(z)$ be bounded and harmonic in R, continuous on C except perhaps in a finite number of points. If the non-euclidean line L in R separates the points of C at which $U(z)$ is positive from the points of C at which $U(z)$ is negative, then no critical point of $U(z)$ lies on L in R. If $U(z)$ is non-negative on C and vanishes on the arc α of C, then the subregion of R bounded by α and by the non-euclidean line joining the end points of α contains no critical points of $U(z)$. This result extends under suitable conditions to critical points of a linear combination with constant coefficients of Green's functions for R and harmonic measures of arcs of C. (Received June 9, 1947.)

332. D. V. Widder and Salomon Bochner: *A homogeneous differential system of infinite order with nonvanishing solution.*

The differential system $(\sin \Pi D)y(x) = \lim_{n\to\infty} \Pi D \prod_{k=1}^{n}(1 - k^{-2}D^2)y(x) = 0$, $h(\pm\infty) = 0$, is homogeneous in the sense that no fundamental solution, e^{kx} $(k = 0, \pm 1, \pm 2, \cdots)$, of the differential equation satisfies the boundary conditions. If the equation were of finite order it could, as a consequence, have no nonvanishing solution. It is shown that every even derivative of the function $k(x) = e^{-x}/(1 + e^{-x})^2$ satisfies the system. Morever, every function $\sum_{k=0}^{\infty} a_k h^{(k)}(x)$ is a solution if $\sum_{k=0}^{\infty} a_k w^k$ is an even entire function of order one and minimal type. A counter example shows that the word "minimal" cannot be replaced by "normal." (Received June 27, 1947.)

Applied Mathematics

333. H. E. Salzer: *Checking and interpolation of functions tabulated at certain irregular logarithmic intervals.*

For functions that are usually represented upon semi-logarithmic graph paper, that is, which behave as polynomials in $\log x$, the problem of checking or interpolation when the x's are in geometric progression is quite simple due to the uniform interval in $\log x$. But in practice functions are often given at some or all of the points 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000 (same as .001, .002, .005, and so on, or .01, .02, and so on). In the present paper coefficients are given which facilitate: (I) checking of such functions when given at some of the more frequently occurring combinations of those points, by obtaining the last divided difference; (II) Lagrangian interpolation according to a generalization of the scheme recently given by W. J. Taylor, Journal of Research, National Bureau of Standards, vol. 35 (1945) pp. 151–155, RP 1667. (Received July 16, 1947.)

334. H. E. Salzer: *Coefficients for expressing the first twenty-four powers in terms of the Legendre polynomials.*

Exact values of the coefficients of $P_m(x)$, the mth Legendre polynomial, in the expression for x^n as a series of Legendre polynomials are tabulated for $n=0, 1, 2, \cdots, 24$. Previous tables due to Byerly or Hobson do not extend beyond $n=8$, and are inadequate for many needs. These coefficients will be useful in approximating a polynomial of high degree (denoted by $f(x)$ after normalization to the interval $[-1, 1]$) by a polynomial of lower degree which will be best in a well known least square sense; that is, for a preassigned r, they will be used to obtain the polynomial $q_r(x)$ of degree not greater than r which minimizes $\int_{-1}^{1}[f(x)-q_r(x)]^2dx$. (Received July 3, 1947.)

335. H. E. Salzer: *Complex interpolation over a square grid, based upon five, six, and seven points.*

Lagrangian coefficients are tabulated for complex interpolation of an analytic function of $Z=x+iy$, which is given over a square grid in the Z-plane. The formulas employed here are based upon the values of the function at five, six, or seven points which are chosen so as to be as close together as possible, at the sacrifice of possible symmetry. (This is a continuation of the tables contained in the article by A. N. Lowan and H. E. Salzer, *Coefficients for interpolation within a square grid in the complex plane*, Journal of Mathematics and Physics vol. 23 (1944), which gives the coefficients for the 3- and 4-point cases.) Denoting the reference point in the lower left-hand corner by Z_0, and h the length of the grid, so that $Z=Z_0+Ph$, where $P=p+iq$, the approximating n-point formulas are of the well known form $\sum L_j^{(n)}(P)f(Z_j)$. Exact values of the coefficients $L_j^{(n)}(P)$ are given for $p=0, .1, .2, \cdots, 1.0$ and $q=0, .1, .2, \cdots, 1.0$. A method for inverse interpolation is indicated, based upon the coefficients of P^m in $L^{(n)}(P)$. (Received July 16, 1947.)