155. Buchin Su: *Descriptive collineations in spaces of K-spreads.*

The author studies descriptive collineations in a space of K-spreads as a generalization of projective collineations in the general geometry of paths. After the equations expressing the conditions for an infinitesimal descriptive collineation which the space of Douglas may admit are derived, the notion of the Lie derivative is utilized in reducing the number of the integrability conditions of these equations. The group property of descriptive collineations is established. (Received December 19, 1946.)

Statistics and Probability

156. P. L. Hsu and H. E. Robbins: *Complete convergence and the law of large numbers.*

A sequence $\{X_n\}$ of random variables is said to converge to 0 *completely* if for every $\epsilon>0$, $\lim_{n\to\infty}[P(|X_n|>\epsilon)+P(|X_{n+1}|>\epsilon)+\cdots]=0$. Let $\{Y_n\}$ be a sequence of independent random variables with the same distribution function $F(y)=P(Y_n\leqq y)$ and such that $\int_{-\infty}^{\infty} ydF(y)=0$, $\int_{-\infty}^{\infty} y^2dF(y)<\infty$. It is proved that the sequence $\{X_n\}=\{n^{-1}(Y_1+\cdots+Y_n)\}$ converges to 0 completely. A partial converse of this theorem is given, and the relation between complete convergence and convergence with probability 1 is clarified. (Received January 18, 1947.)

Topology

157. C. H. Dowker: *An imbedding theorem for paracompact metric spaces.*

A space is called paracompact if every covering by open sets has a neighborhood-finite refinement. A Banach space in which every finite-dimensional linear subspace is Euclidean is called a general Euclidean space or a non-separable Hilbert space or a unitary Banach space. It is shown that a metrizable space is paracompact if and only if it is homeomorphic to a subset of a unitary Banach space. (Received January 31, 1947.)

158. Nathan Jacobson: *On the theory of primitive rings.*

In a previous paper a ring $\mathfrak{A}$ has been called *primitive* if it contains a maximal right ideal $\mathfrak{J}$ whose quotient $(\mathfrak{J}:\mathfrak{A})=0$. Here $(\mathfrak{J}:\mathfrak{A})$ is the totality of elements $b\in\mathfrak{A}$ such that $\mathfrak{A}b\subseteq\mathfrak{J}$. An extrinsic characterization of these rings is that $\mathfrak{A}$ is isomorphic to an irreducible ring of endomorphisms. In this paper the author studies the one-sided ideals in primitive rings and defines certain topologies in $\mathfrak{A}$ by using these ideals. Particular attention is given to the primitive rings that contain minimal ideals. For these, the structure theories given previously for simple rings by the author and by Dieudonné are generalized. (Received December 12, 1946.)

159. Everett Pitcher: *Čech homology invariants of continuous maps.*

The effect of a continuous map on the Čech homology (or cohomology) groups under assumptions appropriate to the theory is stated in terms of exact homomorphism sequences. This is done with the aid of new homology invariants of maps, namely homology groups of the Čech type based on chains of the nerve of the domain which vanish and homology groups based on chains of the nerve of the range which are images. The former are analogous to invariants already introduced by the writer (Bull. Amer. Math. Soc. Abstract 46-5-216) for simplicial maps. The whole development is the geometric extension of the algebraic treatment of homomorphisms of chain complexes in the paper *Exact homomorphism sequences in homology theory* (Bull. Amer. Math. Soc. Abstract 52-1-49) by Kelley and the writer, to appear in Annals of Mathematics. Further developments include duality theorems, necessary and sufficient conditions for isomorphism of the homology groups of domain and range and characterizations of critical levels in critical point theory. (Received January 20, 1947.)

160. J. H. Roberts: *A problem in dimension theory.*

In *Dimension theory* by Hurewicz and Wallman, the following question is raised: If M is a noncompact separable metric space of dimension n, is the set of homeomorphisms of M into I_{2n+1} (cube in $(2n+1)$-euclidean space) a G_δ set in the space of all continuous mappings of M into I_{2n+1}? The present paper answers this question in the negative. In fact examples are given such that the set of homeomorphisms is not even a Suslin set. (Received January 3, 1947.)

161. A. E. Ross: *On continuous transformations of a continuous closed curve.*

Let s be a continuous one-to-one transformation of a closed, but not simply closed, curve with a finite number of points of fixed multiplicity l. Then some power s^k of s has fixed points and every set D_p: p, $s(p)$, $s^2(p)$, $\cdots$ either consists of a finite number of points or has a finite number of limit points. (Received December 9, 1946.)

162. A. E. Ross: *On continuous transformations of a Jordan curve.*

Let s be a continuous one-to-one transformation of a Jordan curve (a continuous simply closed curve) into itself. Consider the set D_p: p, $s(p)$, $s^2(p)$, $\cdots$ of the images of a point p. It is proved that (a) if s^k (k integral and positive) is the smallest power of s having fixed points, then the set D_p has exactly k limit points or consists of exactly k distinct points. If, however, no integral power s^k of s has fixed points then either (b_1) for every p the closure $C[D_p]$ of D_p is the whole Jordan curve or (b_2) there exists a point p for which the closure of D_p is a perfect nowhere dense set, and if q is a point of one of the denumerably infinite intervals not containing the points of $C[D_p]$, then D_q consists only of isolated points. The treatment is elementary throughout. In the cases (b_1) and (b_2) it is shown that there is a p such that D_p is dense in itself. This leads to the classification (b_2) and (b_1) ("transitive case" of van Kampen, Amer. J. Math. vol. 57 (1935) p. 145 and Lewis and Wintner, ibid. vol. 56 (1934)) and relates it to equicontinuity of the set of monotonic functions $t_i(x)$ determining s^i ($i=0, 1, \cdots$) (Received December 9, 1946.)

163. P. A. White: *On the equivalence between avoidability and co-local connectedness.*

Equivalences are found between the avoidability properties defined by R. L. Wilder (*Sets which satisfy certain avoidability conditions*, Casopis pro Pestovani Matematiky a Fysiky vol. 67 (1938) pp. 185–198) and properties involving co-local-connectedness and local co-betti numbers (see E. G. Begle, *Locally connected spaces and generalized manifolds*, Amer. J. Math. vol. 64 (1942) pp. 553–574). In particular for compact spaces S it is shown that the property of being semi-i-connected $(0\leqq i\leqq n-2)$, $(n-1)-$lc, and completely-i-avoidable $(0\leqq 1\leqq n-2)$ at $p\in S$ is equivalent to being i-lc and i-co-lc $(0\leqq i\leqq n-1)$ at p. Also if $p^n(S)=1$ irreducibly and S is semi-$(n-1)$-connected at $p\in S$, then the properties local-$(n-1)$-avoidability at p and $R_n(p)=1$ are equivalent. (Received December 10, 1946.)