η/a (the ratio of the length of the gap to the diameter of the antenna) and η/l (the ratio of the length of the gap to the length of the antenna). The classical sinusoidal current distribution is obtained in the limiting case where η/a is large and η/l is small. A general method of successive approximations is set up, but no proof of convergence is given. (Received October 31, 1946.)

78. Gabor Szegö: *The capacity of a circular plate-condenser.*

Consider two thin circular discs of radius a with a common axis and at a distance d, $d/a=q$, charged to constant and opposite potentials, $V=\pm V_1$. If the charges are $\pm Q$, respectively, the constant $C=Q/V_1$ is called the capacity of the condenser. G. Kirchoff (*Gesammelte Abhandlungen*, p. 112) gave the following approximate formula for this important quantity: $a^{-1}C=(4q)^{-1}+(4\pi)^{-1}\log(1/q)+\alpha(q)$, $\limsup \alpha(q)\leqq(4\pi)^{-1}\cdot(\log(16\pi)-1)=K$ as $q\to 0$. Recently (Acad. des Sciences l'URSS, 1932) Ignatowsky gave the following sharper result: $\lim \alpha(q)=(4\pi)^{-1}(\log 8-1/2)=I$. The proofs are in both cases somewhat incomplete. In the present paper Kirchoff's proof is revised by using Dirchlet's principle. Moreover by means of the so-called Thomson principle a very simple proof is given for $\liminf \alpha(q)\geqq I$. Finally the case of thick plates is discussed. (Received November 23, 1946.)

79. H. L. Turrittin: *Stokes multipliers for asymptotic solutions of a certain differential equation.*

If v is a positive integer, the differential equation $d^ny/dx^n-x^vy=0$, $n\geqq 2$, has n independent solutions $y_j=x^j(1+a_{1j}x^p+a_{2j}x^{2p}+\cdots+a_{mj}x^{mp}+\cdots)$, $p=v+n$, convergent for all x. If the complex x-plane, $x=re^{i\theta}$, is divided into $2p$ sectors by the radial lines $\theta=h\pi/p$, $h=0, 1, \cdots$, Trjitzinsky (Acta Math. (1934) pp. 167–226) has shown that to each sector there corresponds n independent solutions $\tilde{y}_k\sim\xi_k^{v(1-n)/2p}\exp\xi_k\{1+b_1/\xi_k+b_2/\xi_k^2+\cdots\}$ where $\xi_k=(n/p)x^{p/n}e^{2\pi ik/n}$. These asymptotic representations are valid *uniformly* throughout the sector (edges included). Therefore there exists a nonsingular linear relationship $y_j=\sum_{k=0}^{n-1}c_{jk}\tilde{y}_k$, $j=0, 1, \cdots, n-1$. These constants c_{jk}, which change from sector to sector, are the *Stokes multipliers* that have been computed. To do so the author borrowed heavily from the Ford-Newsom-Hughes theory of asymptotic expansion (Bull. Amer. Math. Soc. vol. 51 (1945) pp. 456–461). However this theory does not yield directly the desired uniform asymptotic representation in all cases, nor even the desired form when the real part of ξ_k is negative. The F–N–H theory is extended to supply the requisite information. Scheffé (Trans. Amer. Math. Soc. vol. 40 (1936) pp. 127–154) computed two of the n multipliers corresponding to each j. (Received October 7, 1946.)

Geometry

80. L. M. Blumenthal: *Superposability in elliptic space.* II.

Let f denote a one-to-one correspondence between the points of two subsets P, Q of the elliptic space $E_{n,r}$. Two corresponding subsets A_P, B_Q of P, Q, respectively, are called f-superposable provided there exists a congruence Γ of $E_{n,r}$ with itself which gives the same correspondence between A_P and B_Q as f does. The writer defines a space to have superposability order σ provided any two subsets of the space are superposable whenever a one-to-one correspondence f between the points of the subsets exists such that each two corresponding σ-tuples are f-superposable. A principal result of this paper is that $E_{n,r}$ has minimum superposability order $n+1$. Two subsets of $E_{n,r}$ in a one-to-one correspondence f, which are not f-superposable though each two corresponding n-tuples are f-superposable are called pseudo f-superposable sets of degree n. A metric characterization of such sets of $n+1$ points is given. Whether pseudo f-superposable sets of degree n necessarily consist of exactly $n+1$ points has not yet been determined for $n>2$. (Received October 24, 1946.)

81. S. C. Chang: *Contributions to projective theory of singular points of space curves.*

In this paper a covariant coordinate system for a space curve at an inflection point is described in terms of configurations depending on neighborhoods of order no greater than eight. The principal configurations used are the quadric surface having eighth order contact with the curve at the inflection point and the family of uniplanar cubic surfaces having second order contact with the quadric and eighth order contact with the curve. It is shown that the uniplanar points of all the cubic surfaces lie on the quadric. (Received November 29, 1946.)

82. P. B. Johnson: *A contribution to parallel displacement theory in Riemannian space.*

The equations of Levi-Civita parallel displacement are integrated by means of the matrizant functional. The results in A. D. Michal, *An existence and uniqueness theorem for a nonlinear differential equation in Banach spaces* (Bull. Amer. Math. Soc. Abstract 53-1-39) are applied. The displaced vector is given as a series of terms linear in the undisplaced vector and of increasing order in the components of the directrix. The directrix is the curve along which the vector is displaced with components $x^i=x^i(t)$. The results are applied to the special case of the closed directrix. The change in the displaced vector caused by shifting the directrix is discussed. It is shown that the 1923 result of Paul Dienes giving under certain conditions the first approximation to the change in the displaced vector can be written in matrix form $\delta' V_1=\int_0^1\Omega_t^1 R_{ab}(dx^a/dt)\zeta^b\Omega_0^t dt\, V_0$ (where R_{ab} is the Riemann tensor, Ω_ξ^n is the matrizant functional, ζ^b is the change in the directrix) and is indeed the first Fréchet differential of V with increment ζ^b. The higher order Fréchet differentials with increments ζ^b are discussed. The results are specialized for closed directrices. (Received October 24, 1946.)

83. Edward Kasner and John DeCicco: *A partial differential equation of fourth order connected with rational functions of a complex variable.*

A certain partial differential equation of fourth order is obtained of which any polynomial solution $\phi(x, y)$ represents the numerator of the real part (when expressed as the quotient of two relatively prime polynomials) of a rational function of a complex variable. This equation states that $\log[\phi(\phi_{xx}+\phi_{yy})-(\phi_x+\phi_y)]$ is a harmonic function of (x, y). This new class of rational fractional potential polynomials contains the class of harmonic polynomials as a proper subset. The totality of analytic solutions of this partial differential equation of fourth order consists of the real parts of the special class of polygenic functions defined as the product of an analytic function of $z=x+iy$ by another independent analytic function of the conjugate complex variable $\bar{z}=x-iy$. For this class of polygenic functions, the analogues of the Cauchy-Riemann equations are discussed. (Received October 23, 1946.)

84. Edward Kasner and John DeCicco: *Rational harmonic curves.*

A rational harmonic curve is defined by setting the real part of a rational function of $x+iy$, of degree r, equal to zero. Any such curve is given by $P(x, y)=0$, where P is a polynomial of (x, y) of degree $2r-k$, where $0 \leqq k \leqq r$. The curve has k real asymptotes, all of which pass through a fixed point and which make equal angles with each other. The angle between consecutive asymptotes is π/k. The remaining asymptotes are minimal and $2(r-k)$ in number. The theorem generalizes to rational harmonic curves a theorem of Briot and Bouquet concerning the asymptotes of polynomial harmonic curves, which are defined by setting the real part of any rational integral function of a complex variable equal to zero. Other properties are given by means of the foci of systems of confocal curves. Additional results are found about satellites. (Received October 4, 1946.)

85. C. E. Springer: *Union torsion of a curve on a surface.*

The geodesic torsion at a point of a curve on a surface is the torsion of the geodesic which is tangent to the curve at the point. In this paper the union torsion at a point of a curve C on a surface is defined as the torsion of the union curve in the direction of the curve C, the union curve being defined relative to a given rectilinear congruence. The union torsion is given by a formula which reduces to the expression for geodesic torsion in case the congruence is normal to the surface. It is shown that a union curve is a plane curve if, and only if, it is tangent to a curve of intersection of a developable of the congruence with the surface. (Received October 3, 1946.)

86. A. E. Taylor: *A geometric theorem and its application to biorthogonal systems.*

Let S be a bounded and closed point set in E_n (Euclidean space of n dimensions). Let O be a point such that O and S together are not contained in any subspace of $n-1$ dimensions (such a subspace is hereafter called a plane). Then there exist n linearly independent vectors $x_1, \cdots, x_n$ emanating from O, with terminal points $P_1, \cdots, P_n$ in S, and n planes $p_1, \cdots, p_n$ satisfying the following conditions: (a) p_i contains P_i; (b) p_i is parallel to the plane determined by $x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n$; (c) S and O both lie in the same one of the two closed half-spaces into which p_i divides E_n. This theorem is proved, and then applied to demonstrate the following: If Y_n is an n-dimensional subspace of a normed linear space X, there exist n elements $x_1, \cdots, x_n$ of unit norm in Y_n and n linear functionals $f_1, \cdots, f_n$ of unit norm defined on X, such that $f_i(x_j)=\delta_{ij}$. (Received October 25, 1946.)

Statistics and Probability

87. G. E. Forsythe: *On Nörlund summability of random variables to zero.*

The present paper is an incomplete extension to regular Nörlund summability methods of some previous results (G. E. Forsythe, Duke Math. J. vol. 10 (1943) pp. 397–428, §5) on Cesàro summability in probability of random variables to zero. Corresponding to a sequence p_0 $(=1)$, $p_1, p_2, \cdots$ of non-negative constants, the Nörlund method N_p is defined by the triangular Toeplitz matrix $\|a_{nk}\|$, where $a_{nk} = p_{n-k}(\sum_{k=0}^{n} p_k)^{-1}$. It is conjectured that if $N_p \subset N_q$ with respect to the summability of