Matematisk-Naturvidenskabelig Klasse, no. 10) may be restated as follows. Given an arbitrary finite set of symbols, with A's and B's arbitrarily given strings (zeichenreihen) involving no other symbols than those in the given set, P and Q operational variables, to determine whether B is an assertion in the system with initial assertion A and operations $P\,A_iQ$ *produces* $P\,B_iQ$, $P\,B_iQ$ *produces* $P\,A_iQ$, $i=1, 2, \cdots, \mu$. Through the intermediary of the Turing machine, a known recursively unsolvable decision problem is reduced to the decision problem of a system with initial assertion A' and operations $P\,A_i'\,Q$ *produces* $P\,B_i'\,Q$, $i=1, 2, \cdots, \mu'$, having the property that the set of assertions of the system is unchanged when the system is transformed into Thue type by adding the inverse operations $P\,B_i'\,Q$ *produces* $P\,A_i'\,Q$. The recursive unsolvability of the problem of Thue easily follows. (Received September 20, 1946.)

Statistics and Probability

395. H. W. Becker: *Rooks and rhymes.*

Kaplansky and Riordan have shown that ${}_cR_{r-1}$, or the number of ways of putting c non-attacking rooks on a right-angled isosceles triangle of side $r-1$, is the Stirling number $\Delta^{r-c}0^r/(r-c)!$. This is the number of selections of c points on such a chess board, such that none have any row or column index in common, an idea incidental to various statistical problems. The point sets are well ordered, in 1-to-1 correspondence with the sequations (rhyme schemes) and distribution cycles. Further classifications ${}^{(q)}R_r$, ${}^{q}R_r$, ${}_{c'}R_r$, and ${}_{(c)}R_r$ are formulated in terms of rhyme functions according to: row location of topmost rook; number of rooks in the principal diagonal; column vacancies; and column location of the bottom rook. A typical isomorphism is ${}_{c'}R_r$ is equal to the R_r with cth column empty is equal to the number of distributions of $r+1$ men into crews, such that one man is incompatible with, and must be segregated from, $r-c$ other men. (Received July 24, 1946.)

396. Nilan Norris: *An extension of an equality among averages.*

A classic theorem of algbra states that if A, G, and H are respectively the arithmetic, geometric, and harmonic means of two positive real numbers, then $G^2=AH$. In this paper proof is given that a sufficient condition for the extension of this equality to any n positive numbers is that the logarithms of the variates be symmetrically distributed about an axis of ordinates at $\log G$ of the n numbers. For samples and populations obeying the symmetry condition with respect to $\log G$, the theorem is extended to an unlimited number of averages as yielded by certain sample and integral forms of general means (generalized means value functions). (Cf. an unpublished manuscript of J. B. Canning, *A theorem concerning a certain family of averages of a certain type of frequency distribution.*) (Received August 15, 1946.)

Topology

397. Salomon Bochner and Deane Montgomery: *Groups on analytic manifolds.*

This paper studies the nature of complex and real Lie groups acting on certain complex or real manifolds in the large. For example, it proves that the group of all complex analytic homeomorphisms of a compact complex manifold is a complex Lie group. (Received September 20, 1946.)

398. C. H. Dowker: *An extension of Alexandroff's mapping theorem.*

A continuous mapping of a space into the nerve of an open covering is called canonical if the inverse image of the star of each vertex of the nerve is contained in the open set which corresponds to that vertex. An existence theorem for such mappings was proved by P. Alexandroff (Ann. of Math. (2) vol. 30 (1928) p. 121) and extended by S. Lefschetz (*Topics in topology*, Princeton, 1942, pp. 35–49). Another extension is the following: A normal space has a canonical mapping into the naturally topologized nerve of a covering if and only if the covering has a neighborhood-finite refinement. Consequently, a space has a canonical mapping into the naturally topologized nerve of every covering if and only if the space is paracompact and normal. The proof depends on showing that every complex with natural topology is a paracompact space. (Received October 1, 1946.)

399. Samuel Eilenberg and Saunders MacLane: *Determination of the second homology and cohomology groups of a space by means of homotopy invariants.*

The homotopy invariants of a topological space X include the fundamental group π_1, the second homotopy group π_2, and the fashion in which π_1 operates on π_2. A new invariant k^3 of the space X is introduced as a 3-dimensional (algebraic) cohomology class of π_1 with coefficients in π_2. By using this additional invariant, the second cohomology groups of the space can be constructed algebraically from π_1 and π_2. This includes the known determination of the subgroup of spherical annihilators as the second (algebraic) cohomology group of π_1. The homology groups are obtained by duality. If $k^3=0$, then the spherical subgroup of the homology group is a direct summand of the whole group. (Received August 23, 1946.)

400. Ky Fan: *On partially ordered additive groups of continuous functions.*

The totality $C(\Omega)$ of all real bounded continuous functions defined on a topological space Ω may be regarded as a *partially ordered additive group* (p.o.a.g.). An "*ideal space*" Ξ is constructed from the p.o.a.g. $C(\Omega)$. If Ω is a compact Hausdorff space, Ω and Ξ are homeomorphic. This provides a simple proof of the following theorem due to S. Kakutani (Ann. of Math. vol. 42 (1941) p. 1008), M. Krein and S. Krein (Rec. Math. (Mat. Sbornik) N.S. vol. 13 (1943) p. 25): If Ω and Ω^* are two compact Hausdorff spaces (or if they are completely regular and satisfy the first countability axiom), Ω and Ω^* are homeomorphic if and only if $C(\Omega)$ and $C(\Omega^*)$ are isomorphic as p.o.a.g. More precisely, the general form of the isomorphisms (in the sense of p.o.a.g.) between $C(\Omega)$ and $C(\Omega^*)$ is obtained. For any topological space Ω, to every "*order-preserving direct sum decomposition*" of the p.o.a.g. $C(\Omega)$ into two proper subgroups, there corresponds a decomposition of Ω into two disjoint closed proper subsets. A characterization of the dimension of a compact metric space Ω in terms of properties of the p.o.a.g. $C(\Omega)$ is also given. (Received September 16, 1946.)

401. R. H. Fox: *On a problem of S. Ulam concerning Cartesian products.*

Let A be the Cartesian product of a line segment with the bounded 3-dimensional manifold which is obtained from the lens-space (5, 1) by removing a small 3-cell, and denote by B the bounded 4-dimensional manifold which is similarly obtained from the lens-space (5, 2). It can be shown that A and B are not homeomorphic and that $A\times A$ is homeomorphic to $B\times B$. This answers in the negative a question proposed by S. Ulam (Fund. Math. vol. 20 (1933)). (Received September 24, 1946.)

402. S. T. Hu: *On extension of homotopy.*

A closed subset X_0 of a topological space X is said to have the absolute homotopy extension property AHEP if, for an arbitrary topological space Y, every partial homotopy $f_t(X_0)\subset Y$ of an arbitrary mapping $f_0(X)\subset Y$ has an extension $f_t^*(X)\subset Y$ such that $f_0^*=f_0$. It is well known that X_0 has the AHEP in X if X is a polyhedron and X_0 a subpolyhedron (Alexandroff-Hopf, p. 501). The author gives several generalizations of this well known fact. (1) If X_0 and X are compact ANR, then X_0 has the AHEP in X. (2) If $T=(X\times 0)+(X_0\times I)$ is an ANR, then X_0 has the AHEP in X. (3) If X is a compact ANR, then X_0 has the AHEP in X if and only if T is an ANR. The span $s(f_t, X)$ of a homotopy $f_t(X)\subset Y$ over X is defined to be the least upper bound of $\rho(f_{t_1}(x), f_{t_2}(x))$ for all $x\subset X$ and $0\leqq t_1<t_2\leqq 1$. For each $\epsilon>0$, every partial homotopy $f_t(X_0)\subset Y$ of an arbitrary mapping $f_0(X)\subset Y$ has an extension $f_t^*(X)\subset Y$ such that $f_0^*=f_0$ and $s(f_t^*, X)\leqq s(f_t, X_0)+\epsilon$, provided that either of the following conditions is satisfied: (i) Y is an ANR and X a normal space; (ii) Y is a metric space and X, X_0 are compact ANR. (Received August 29, 1946.)

403. Deane Montgomery: *A theorem on locally euclidean groups.*

It is proved that if G is a locally euclidean, connected, simply connected topological group of dimension n greater than one, then G contains a closed subgroup of dimension greater than zero and less than n. (Received August 10, 1946.)

404. S. B. Myers: *Banach spaces of continuous functions.*

If B is a Banach space, let T denote any maximal subset of B with the property that, for any finite subset $(t_1, \cdots, t_n)$, $\|\sum t_i\|=\sum\|t_i\|$, and let F_T be the functional $F_T(b)=\inf_{t\in T}(\|b+t\|-\|t\|)$. If X is a topological space, $B(X)$ will denote the space of all real bounded continuous functions on X. A subset $E\subset B(X)$ is *completely regular* over X if for each closed set $C\subset X$ and point $x\in\epsilon X-C$, there exists $b\in E$ such that $b(x_0)=\|b\|$, $\sup_{x\in C}|b(x)|<\|b\|$. The following two theorems are the main results of this paper, the first being a generalization of the Banach-Stone theorem, and the second a characterization of the space of continuous functions. I. If X_1, X_2 are compact, and if a closed, completely regular, linear subspace of $B(X_1)$ is equivalent to such a subspace of $B(X_2)$, then X_1 is homeomorphic to X_2. II. A Banach space is equivalent to $B(X)$ for some compact X if and only if the following conditions are satisfied: (a) every F_T is linear, (b) B contains an element e such that every $b\in B$ satisfies either $\|b+e\|=\|b\|+1$ or $\|b-e\|=\|b\|+1$, (c) for every $b\in B$ there exists $b'\in B$ such that for all F_T for which $F_T(e)=1$, $F_T(b')=|F_T(b)|$. (Received September 27, 1946.)

405. Everett Pitcher: *Critical point inequalities for a nondegenerate functional.*

A simplified proof is presented of the inequalities of Morse for the case of a positive regular integral of the calculus of variations on curves joining two fixed points on a manifold in the nondegenerate case for which the two points are conjugate on no extremal. Noteworthy features are the use of singular homology, the avoidance of double neighborhood definitions and techniques and of the concept of upper reducibility and the construction of deformations which carry cycles to their inferior limits. If J_c^- denotes the set of curves for which $J<c$ and if g is an extremal on which $J=c$, then the extremal is characterized by the relative homology groups $H_r(J_c^- \cup g \bmod J_c^-)$. The method is related to ideas in the last section of a paper by Morse on functional topology (Ann. of Math. vol. 38 (1937) pp. 386-449). Received September 28, 1946.)

406. Everett Pitcher: *Exact homomorphism sequences for a nest of three spaces.*

The basic construction of an exact homomorphism sequence is set up for relative homology theory of a nest of three chain complexes and for a nest of three sets in the singular and Čech theories. The duality on homology and cohomology sequences is set up in the abstract form and carried over to the two applications. Relative manifold duality is discussed, the duality being between a sequence of relative Čech homology groups for a nest consisting of a manifold and two closed subsets and a sequence of singular homology groups on complementary subsets in complementary dimensions. The work is an extension of a paper with J. L. Kelley titled *Exact homomorphism sequences in homology theory* and announced in Bull. Amer. Math. Soc. Abstracts 52-1-49 and 52-5-209. It is related to the construction of the exact sequence on relative homology groups of three sets as a theorem in the axiomatic homology theory of Eilenberg and Steenrod and to the abstract developments of Mayer. (Received September 28, 1946.)

407. Fred Supnick: *On the problem of Tait.* Preliminary report.

Let an alternating chain of marks, $+$'s and $-$'s, be associated with all the vertices of a polygon, except two which are adjacent. Let some two adjacent marks be transformed in a sense preserving manner along the polygon, to the blank spaces. The problem of Tait deals with the number of such transformations which are needed to separate the $+$'s from the $-$'s (with different signs adjacent to the blank spaces). This problem has been solved by H. Dellanoy (cf. W. Ahrens, *Mathematische Unterhaltungen und Spiele*, vol. 1, p. 19). In this paper the author studies the above problem subject to the restriction that *the pair of marks which are moved in each transformation must be different.* The number of transformations required to achieve separation becomes a function of the number of blank spaces. Formulas are obtained for the least number of blank spaces for which a separation is possible, and for the least number of steps in which the separation can be effected. A method is given for effecting this separation. It is also noted that these problems of Tait might be interpreted as problems of cooperative phenomena where the marks are considered as spins. (Received September 28, 1946.)