preceding abstract (with $\alpha\beta$ replaced by α/β). B_3. Multiplication is commutative. C. The dyads of T, including ω/ξ, form a commutative group under the undefined compostion "addition." Under C postulates are first stated (C_1) in terms of addition of dyads with the same posterior elements (denominators); it is then assumed (C_2) that α/β, γ/δ in T imply the existence of ξ such that $\gamma/\delta=\xi/\beta$. Definition: $\alpha/\beta+\gamma/\delta=\lambda/\mu$ means: There exist ξ, η such that $\gamma/\delta=\xi/\beta$, $\eta/\beta=\lambda/\mu$, where $\alpha/\beta+\xi/\beta=\eta/\beta$. D. Multiplication over addition to the right (left) is distributive. The independence of C_2 is discussed. Application is made to the author's *Foundations of Grassmann's extensive algebra* (Amer. J. Math. vol. 35 (1913) pp. 39–49). (Received September 27, 1946.)

354. J. D. Swift: *Periodic functions over a finite field.*

A function of period a over the Galois field, $GF(p^n)$, is defined as a function over the elements of the field to the elements of the field such that $f(x+na)=f(x)$, where n is an integer. Multiply periodic functions are defined in an analogous manner. A $GF(p^n)$ admits functions with a number of independent periods not greater than $n-1$. The basic function of period a is: $f(a;x)=x^p-a^{p-1}x$. The basic function of periods $a_1, \cdots, a_n$ may be defined recursively from the above as: $f(f(a_1, \cdots, a_{n-1}; a_n); f(a_1, \cdots, a_{n-1}; x))$. These basic functions are odd and additive, and $f(a_1, \cdots, a_n; cx)$ is periodic with periods $a_1/c, \cdots, a_n/c$. The principal result is: *Any periodic function of periods* $a_1, \cdots, a_k$ *over the* $GF(p^n)$, $k\leqq n-1$, *may be expressed by*: $g(x)=\sum_1^l \alpha_i f^i(a_1, \cdots, ak; x)+\alpha_0)$, where $l=p^{n-k}-1$, *and the* α_i *are a set of elements of the field.* (Received September 16, 1946.)

355. P. M. Whitman: *Finite groups with a cyclic group as lattice-homomorph.*

It is shown that if G and H are groups, $L(G)$ and $L(H)$ their lattices of subgroups, $L(G)$ is finite, $L(H)$ is a lattice-homomorphic image of $L(G)$, and H is cyclic, then G contains a cyclic subgroup which is mapped onto H by the homomorphism. (Received September 26, 1946.)

Analysis

356. Warren Ambrose: *Direct sum theorem for Haar measures.*

A variation of a theorem of A. Weil (*L'intégration dans les groupes topologiques*, Paris, 1940, pp. 42–45) is proved. (Received September 19, 1946.)

357. R. F. Arens: *Location of spectra in Banach *-algebras.*

A Banach *-algebra A is a Banach space with a continuous multiplication and a *-operation satisfying $(\lambda f+\mu g)^*=\bar{\lambda}f^*+\bar{\mu}g^*$, $f^{**}=f$, $(fg)^*=g^*f^*$, and $k|f|\,|f^*|\leqq|ff^*|$, $k>0$. One can renorm A such that $\|fg\|\leqq\|f\|\,\|g\|$, and there will exist $k'>0$ such that $k'\|f\|\,\|f^*\|\leqq\|ff^*\|$. It is proved that, if $f=u+iv$, $u^*=u$, $v^*=v$, $uv=vu$, then $|x\cos\theta+y\sin\theta|\leqq\|u\cos\theta+v\sin\theta\|$ for any complex number $x+iy$ in the spectrum of f, and $0\leqq\theta\leqq 2\pi$. Thus if $f=f^*$, the spectrum is real. The case $k'=1$ studied by I. Gelfand and M. Neumark, Rec. Math. (Mat. Sbornik) N. S. vol. 12 (1943) pp. 197–213, is a special case. (Received August 8, 1946.)

358. Lipman Bers: *A property of bounded analytic functions.*

Let $f(z)$ be bounded and analytic for $|z|<1$. If $\{z_n\}$ is a sequence of points such that $z_n \to 1$, $\tan \arg (z_n - 1) = O(1)$, $f(z_n) \to \alpha$, then either $f = \alpha$ at infinitely many points, or a similar sequence exists on any nontangential path terminating at $z = 1$. If in addition $|z_n| \leqq |z_{n+1}|$, $|z_n - z_{n+1}| = O(1 - |z_{n+1}|)$, then either $f = \alpha$ at infinitely many points, or f possesses at $z = 1$ the sectorial limit α. (Received September 12, 1946.)

359. Lipman Bers: *On rings of analytic functions.*

Let D be a plane domain. The regular one-valued analytic functions defined in D form a topological ring $R(D)$. Theorem 1. Two rings $R(D)$ and $R(D')$ are isomorphic if and only if D' can be mapped onto D by a conformal transformation, or by a conformal transformation followed by a reflection. Theorem 2. If D is not the whole Riemann sphere, then any isomorphism between $R(D)$ and $R(D')$ is a homeomorphism due to either a conformal or an anticonformal transformation of D onto D'. (Received September 30, 1946.)

360. R. C. Buck: *Interpolation series.*

Carlson (Nova Acta Regiae Societatis Scientiarum Upsaliensis (4) vol. 4 (1915)) has given conditions on $h(\theta)$, the growth function of the entire function $f(z)$, under which the Newton series $\sum_{n=0}^{\infty} z(z-1) \cdots (z-n+1)\Delta^n f(0)/n!$ converges to $f(z)$. Gontcharoff (Rec. Math. (Mat. Sbornik) vol. 42 (1935) pp. 473–483) has achieved similar results for the Abel series $\sum_{n=0}^{\infty} z(z-n)^{n-1} f^{(n)}(n)/n!$. In the present paper, summability of these and other expansions is considered. In particular, the Newton series is Mittag-Leffler summable for all functions of the class $K(a, c)$, $c < \pi$, thus providing another proof of the familiar Carlson uniqueness theorem. These results are compared with those contained in a paper of Gelfond (Rec. Math. (Mat. Sbornik) N. S. vol. 4 (1938) pp. 115–148). A simple consequence is that if $f(z) \in K(a, c)$, $c < \pi$, and $(-1)^n \Delta^n f(0) \geqq 0$, then for any p, $\lim n^p \Delta^n f(0) = 0$; thus, if $f(n)$ is integral, $f(z)$ is a polynomial. (Received September 27, 1946.)

361. R. C. Buck: *On some properties of arithmetic density.* Preliminary report.

In a previous paper it was shown that if $\{A_k\}$ is a monotone sequence of sets of $\mathcal{D}$, the class of all sets of integers having a density, a definition of $\lim A_k$ can be given, unique to within sets of density zero, and for which density is continuous. In the present paper, this is applied to yield a wide variety of results. A simple example is the following: if $f(z)$ is an entire function of type zero, and $|f(n+1)/f(n)| \geqq \delta > 0$, then $\lim n^{-1} \log |f(n)| = 0$. (Received September 27, 1946.)

362. V. F. Cowling: *A generalization of a theorem of LeRoy and Lindelöf.*

Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ with radius of convergence unity. Let the coefficients a_n be the values taken on by an analytic function $a(z)$ at $z = 0, 1, \cdots$. Suppose $a(z)$ is analytic and single-valued with the possible exception of a pole of order K at infinity in an angle with vertex $h > 0$ (non-integral) on the axis of reals and including the axis of positive reals in its interior. Let the sides of this angle make angles ψ_1 and ψ_2 with the axis of positive reals. Then, if $\gamma > 0$ but otherwise arbitrarily small and $0 < \psi_1 < \pi/2$, $0 < \psi_2 < \pi/2$, $f(z)$ is regular in the domain common to $r \leqq \exp[\theta \tan \psi_1] - \gamma$ and $r \leqq \exp[(\theta - 2\pi) \tan \psi_2] - \gamma$ for $0 \leqq \theta < 2\pi$. A similar theorem is proved in the case that $a(z)$ has an essential singularity at infinity. If the angles ψ_1 and ψ_2 are both greater than 90° in magnitude, some theorems are proved which are closely related to the sufficient condition that a function defined by a Taylor series have exactly one singularity in the entire complex plane. (Received September 14, 1946.)

363. V. F. Cowling: *On the form of certain entire functions.*

Let $F(z)=\sum_{n=0}^{\infty}(a_n/n!)z^n$, where the a_n are integers. If $F(z)$ is an entire function not a polynomial and $\lim\sup_{r\to\infty} \log |F(re^{i\theta})|/r=\rho(\theta)<1+\cos\theta$ for $0\leqq\theta\leqq 2\pi$ then $F(z)=Q(z)e^z$, where $Q(z)$ is a polynomial in z. Other theorems of a similar nature are proved. (Received September 14, 1946.)

364. C. L. Dolph: *Nonlinear integral equations of the Hammerstein type.*

The author generalizes certain results of A. Hammerstein (Acta Math. vol. 54 (1930)), R. Iglisch (Math. Ann. vol. 108 (1933)), and M. Golomb (Math. Zeit. vol. 39 (1935)) on the existence of solutions to the nonlinear integral equation $\psi(x)=\int_a^b k(x, y)f[y, \psi(y)]dy$. A solution will exist in L_2 if the linear operator is Hermitian and if $f(x, y)$ satisfies $\mu_N y+A\leqq f(x, y)\leqq \mu_{N+1}y+A$ for large $|y|$, $\lambda_N<\mu_N<\mu_{N+1}<\lambda_{N+1}$, λ_N and λ_{N+1} being the Nth and $(N+1)$th characteristic values of the kernel. The proof is by the method of Leray-Schauder. The solution is unique and a constructive process is given for it when $\partial f(x, y)/\partial y$ satisfies the derivative of the above inequality. When the kernel is positive-definite and $f(x, y)$ satisfies $\mu_N y^2-C_N\leqq 2\int_0^y f(x, v)dv\leqq \mu_{N+1}y^2+C_{N+1}$ the equation is equivalent to an Euler-Lagrange equation of a functional over L_2. Although under these conditions no a priori estimate can exist solely for topological reasons, the calculus of variations in the large establishes the existence of a solution when the associated functional possesses only one maximum on each linear manifold in L_2 parallel to the one spanning the origin and the first N characteristic functions of the kernel. (Received August 23, 1946.)

365. W. F. Eberlein: *Weak compactness in Banach spaces.*

Let E be any Banach space and denote by E_T the linear topological space formed by the elements of E under the weak neighborhood topology. A set S in E_T is said to be compact if every infinite subset of S possesses at least one limit point in E_T. (As shown by Šmulian, this type of compactness is equivalent to weak sequential compactness.) It is proved that a set S in E_T is bicompact if and only if it is closed and compact. This result contains a theorem previously announced by the author (cf. Bull. Amer. Math. Soc. Abstract 52-7-231), as well as a demonstration of the equivalence of various notions of "weak" and "topological" completeness introduced by others. (Received August 5, 1946.)

366. Alfred Horn: *The asymptotic behavior of solutions of systems of Volterra integral equations.*

Previously announced results (Bull. Amer. Math. Soc. Abstract 52-5-154) are extended and generalized to systems. Using capitals for n-rowed square matrices and lower case for vectors, consider the system $u(x, y, \lambda)=\lambda\int_y^x K(x, t, \lambda)u(t, y, \lambda)dt+f(x, y, \lambda)$, where K and f have asymptotic developments of the form $\sum_{i=0}\lambda^{-i}K^i$, $\sum_{i=0}\lambda^{-i}f^i$. Suppose the characteristic roots $\gamma_j(x)$ of $K^0(x, x)$ are distinct and not 0 for each x in $[\alpha, \beta]$. Let Z be the region in which $Rl(\lambda\gamma_1(x))\geqq Rl(\lambda\gamma_j(x))$ for $\alpha\leqq x\leqq\beta$, and let Z_1, Z_2 be the subregions in which $Rl(\lambda\gamma_1(x))\leqq 0$, and $Rl(\lambda\gamma_1(x))\geqq 0$ respectively. Then u has an asymptotic expansion $\sum_{i=0}\lambda^{-i}F^i(x, y)e(x, y)+\sum_{i=0}\lambda^{-i}g^i(x, y)$ in Z, where $e_j(x, y)=\exp(\lambda\int_y^x\gamma_j(s)ds)$, the remainder being bounded in Z_1 and of order $\exp(\lambda\int_y^x\gamma_1(s)ds)$ in Z_2. The assumption $\gamma_j(x)\neq 0$ is essential here and for this case the above theorem constitutes a generalization of G. D. Birkhoff's theorem for systems of differential equations. Better bounds are obtained for the subregions in which $|\pi-\arg\lambda\gamma_1|\leqq\pi/2-\delta$, $|\arg\lambda\gamma_1|\leqq\pi/2-\delta$. When f is the pth column of K, u becomes the pth column of the resolvent S of K. In this case, $g^1\equiv 0$, and $g^2\equiv\cdots\equiv g^r\equiv 0$ for all the columns of S if and only if there exist matrices A^i, B^i such that $K^i(x, y)=\sum_{j=0}^{r-2}A^j(x)B^{r-2-j}(y)$, $0\leqq i\leqq r-2$. Applications are given. (Received July 29, 1946.)

367. W. N. Huff: *On the type of the polynomials generated oy $f(x\tau)\phi(\tau)$.*

The polynomials $y_n(x)$ generated by $g(x,\tau)=f(x\ \tau)\phi(\tau)=\sum_0^\infty y_n(x)\tau^n$ with $f(x\tau)=\sum_0^\infty a_n(x\tau)^n/n!$ and $\phi(\tau)=\sum_0^\infty b_n\tau^n/n!$ are considered. Necessary and sufficient conditions for the polynomials to be of A, B and C type k and of infinite type in accordance with the definition of Sheffer are obtained along with the explicit relations satisfied by the polynomials in each case. The Hermite polynomials are the only orthogonal polynomials of type zero among the $y_n(x)$. For polynomials of A type 1 the differential equation and recurrence relation are found. The $y_n(x)$ include no orthogonal polynomials of A type 2 and the orthogonal polynomials of A type 1 are those of Laguerre. (Received September 19, 1946.)

368. R. C. James: *Inner products in normed linear spaces.*

Let $x\perp y$ be defined as meaning $\|x+ky\|\geqq\|x\|$ for all k. If $x\perp y$ implies $y\perp x$, or if $y\perp x$ and $z\perp x$ imply $y+z\perp x$, then the norm can be defined by an inner product if the space is of three or more dimensions. However, $x\perp y$ and $x\perp z$ imply $x\perp y+z$ if and only if the unit sphere has a tangent hyperplane at each point. It then follows from the relations between orthogonality and linear functionals that the norm can be defined by an inner product if the space is of three or more dimensions and one of the following conditions is satisfied: (1) Whenever x and y are such that there is a nonzero linear functional f with $f(x)=\|f\|\cdot\|x\|$ and $f(y)=0$, then there is a nonzero linear functional g with $g(y)=\|g\|\cdot\|y\|$ and $g(x)=0$. (2) Whenever $f(x)=\|f\|\cdot\|x\|$ and $g(y)=\|g\|\cdot\|y\|$, there are numbers a and b (not both zero) such that $f(ax+by)+g(ax+by)=\|f+g\|\cdot\|ax+by\|$. (3) Normal projection is a linear operation if defined, where the normal projection of x on a linear subset L is the element $u\in L$ for which $\|x-u\|$ is minimum. (Received September 25, 1946.)

369. M. Z. Krzywoblocki: *A local maximum property of the fourth coefficient of schlicht functions.*

Let the power series $w=f(Z)=Z+a_2Z^2+a_3Z^3+a_4Z^4+\cdots$ define a one-to-one conformal mapping of the unit circle $|Z|<1$ onto some region in the w-plane. As is well known, Faber and Bieberbach proved that $|a_2|\leqq 2$. Löwner showed that $|a_3|\leqq 3$. Schaeffer and Spencer gave a second proof that $|a_3|\leqq 3$. In the present paper it is shown that $|a_4|$ has a local maximum for the function $f(Z)=Z/(1-e^{i\phi}Z)^2=Z+2e^{i\phi}Z^2+3e^{2i\phi}Z^3+4e^{3i\phi}Z^4+\cdots$, thus completing certain calculations indicated by Yoh. More precisely, the following theorem was proved: If Löwner's K-function has the form $K(t)=e^{i[\alpha_0+\alpha(t)]}$ where α_0 is an arbitrary real constant and

$\alpha(t)$ is a real function satisfying the condition $|\alpha(t)| \leqq 1/18000$, then for the corresponding fourth coefficient b_4, $|b_4| \leqq 4$. No importance should be attached to the value 1/18000; it can easily be replaced by a somewhat larger number merely by refinement of the estimates given below. It would be desirable, however, to prove the theorem under the weaker restriction that $\int_0^\infty \alpha(t)^2 dt < \eta_0$. (Received September 12, 1946.)

370. G. W. Mackey: *On the domains of closed linear transformations in Hilbert space.* Preliminary report.

The principal result of this paper is the following. Let $\mathfrak{M}$ be the family of all closed linear subspaces of Hilbert space. Let $\overline{\mathfrak{M}}$ be the smallest family of linear subspaces of Hilbert space which includes $\mathfrak{M}$ and contains with each two subspaces A and B their intersection $A \cap B$ and their linear union $A \dotplus B$. Then a subspace is in $\overline{\mathfrak{M}}$ if and only if it is the domain of definition of a closed linear transformation. Furthermore, every member of $\overline{\mathfrak{M}}$ may be represented in the form $(A \dotplus B) \cap (C \dotplus D)$ where A, B, C and D are members of $\mathfrak{M}$. (Received September 30, 1946.)

371. A. P. Morse: *Perfect blankets.*

In this paper, which is to appear in Trans. Amer. Math. Soc., it is shown that a rather wide class of Euclidean closed sets can be so associated with a point that if f and ϕ are *any* two measures of a quite general sort then the Euclidean closed sets can be used in forming difference quotients $f(\beta)/\phi(\beta)$ which, in the limit, determine a derivative almost everywhere in the sense of ϕ. (Received August 30, 1946.)

372. P. V. Reichelderfer: *On the definition of the essential multiplicity for continuous transformations in the plane.*

Let $T: z = t(w)$, $w \in S$, be a bounded continuous transformation from the set S in the w-plane into the z-plane. In the paper by Radó and Reichelderfer entitled *A theory of absolutely continuous transformations in the plane* (Trans. Amer. Math. Soc. vol. 49 (1941) pp. 258–307) different definitions are used for the essential multiplicity of a point z under T in S, according as S is a bounded finitely connected Jordan region or a bounded domain. The main purpose of this note is to show that their definitions differ merely in form by proving the following theorem: Given a bounded continuous transformation $T: z = t(w)$, $w \in D$, defined on a bounded domain D in the w-plane, a positive number ϵ, and a point z in the z-plane, there exists a bounded continuous transformation $T_\#: z = t_\#(w)$, $w \in D$, whose distance from T on D is less than ϵ, and such that the number of models of z under $T_\#$ in D is equal to the essential multiplicity of z under T in D. (Received August 7, 1946.)

373. P. C. Rosenbloom: *Entire associative functions.*

If $f(z_1, z_2)$ is an entire function of two complex variables and $f(z_1, f(z_2, z_3)) \equiv f(f(z_1, z_2), z_3)$, then f is one of the functions c, z_1, z_2, z_1+z_2-a, $(z_1-a)(z_2-a)/b$ where a, b, and c are constants and $b \neq 0$. The proof depends on repeated use of Picard's theorem for functions of one complex variable. (Received September 30, 1946.)

374. W. T. Scott and H. S. Wall: *On the convergence and divergence of continued fractions.*

The authors show that a necessary condition for the convergence of the continued fraction $K_1^\infty(1/b_p)$ is that at least one of the following three statements holds: (a) $\sum |b_{2p+1}|$ diverges, (b) $\sum |b_{2p+1}s_p^2|$ diverges, where $s_p = b_2 + b_4 + \cdots + b_{2p}$, (c) $\lim_{p\to\infty} s_p = \infty$. This condition, which first appeared in a theorem of Hamburger, is called condition (H). In particular, they show that the continued fraction diverges if $\sum b_{2p}$, $\sum b_{2p+1}$ converge, at least one absolutely, thus extending a result of Stern and von Koch. The condition (H) is sufficient for convergence in the case where $b_{2p-1} = k_{2p-1}z_p$, $b_{2p} = k_{2p}$, $k_1 > 0$, $k_{2p+1} \geqq 0$, $R(k_{2p}) \geqq 0$, $R(z_p) \geqq \delta$, $|z_p| < M$ ($\delta > 0$, $M > 0$, $p = 1, 2, \cdots$). This result includes theorems of Stieltjes, Van Vleck, Hamburger and Mall. (Received August 19, 1946.)

375. I. E. Segal: *The group algebra of a locally compact group.*

Earlier results of the author (see Bull. Amer. Math. Soc. Abstract 46-7-366 and Proc. Nat. Acad. Sci. U.S.A. vol. 27 (1940) pp. 348–352) are extended and refined. (Received August 10, 1946.)

376. J. E. Wilkins: *The converse of a theorem of Tchaplygin on differential inequalities.*

If $y(x)$ is a solution of the equation $L[y] \equiv y'' - p_1y' - p_2y - q = 0$, such that $y(x_0) = y_0$, $y'(x_0) = y_0'$, and if $v(x)$ is such that $L[v] > 0$, $v(x_0) = y_0$, $v'(x_0) = y_0'$, then $v(x) > y(x)$ when $x_0 < x \leqq x_1$ provided that x_1 is the first zero to the right of x_0 of the solution $u(x)$ of the equations $u'' - p_1u' - p_2u = 0$, $u(x_0) = 0$, $u'(x_0) = 1$. This is a best possible result in the sense that either x_1 does not exist or there exists a function $v(x)$ satisfying the above requirements for which $v(x) - y(x)$ vanishes at a point arbitrarily close to x_1. (Received September 27, 1946.)

Applied Mathematics

377. H. W. Becker: *Circuit algebra.*

By means of the symbols $+$, $\|$, and $\perp$, any passive electrical network is representable on the linotype. They denote series, parallel, and bridge connections respectively, and have inverses $-$, $\_$, and $\top$. The definition $R = a\|b = ab/(a+b)$ generates a system parallel to ordinary arithmetic, except that infinity and zero exchange roles, and so on, hence called paraarithmetic. The number of integer solutions of this equation depends only on the prime factor structure, not magnitude, of R. If $R = p_1^{n_1} \cdots p_m^{n_m}$, this number is $\Psi_m(C+1)$, where $C_0 = 1$, $C_v = 2^{v-1}$, and Ψ is an expansion with generalized binomial coefficients (m, v), the sum of the products of the n's v at a time. Where all the n's equal 1, this reduces to $(3^m+1)/2$. Considered as a static structure, a *SP* network is collapsible. Rigidity is imparted by bridge connections, or trusses. The elementary model is the Wheatstone bridge $(a_1+a_2)\|(b_1+b_2)\perp\beta = [(a_1+a_2)\cdot(b_1+b_2)\beta + a_1a_2(b_1+b_2) + (a_1+a_2)b_1b_2]/[(a_1+a_2+b_1+b_2)\beta + (a_1+b_1)(a_2+b_2)]$. In this algebra, shorting or opening an operand effects remarkable transformations amongst the operators; as, $(a_1+a_2)\|(b_1+b_2)\perp 0 = a_1\|b_1 + a_2\|b_2$. In general, $\perp$ connections are specified by subscripts at the pluses involved. Thus the network whose components are the edges of a cubic lattice energized at two opposite vertices is $[(a_1 +_1 a_2)\|(b_1 +_2 b_2) + a_3]\|[c_1 + (c_2 +_1 c_3)\|(d_1 +_2 d_2)]\perp\beta_1\perp\beta_2$. By a method of combinatory synthesis, the total and transfer conductances are then formulated, alternatively to the Kirchhoff method. (Received September 9, 1946.)