ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

347. A. T. Brauer: *Limits for the characteristic roots of a matrix.* II.

Let $A=(a_{\kappa\lambda})$ be a square matrix of order n. Set $P_\kappa=\sum_{\lambda=1,\lambda\neq\kappa}^{n}|a_{\kappa\lambda}|$. In an earlier paper (Duke Math. J. vol. 13 (1946)) it was proved that each characteristic root ω_ν of A must lie in at least one of the n circles $|z-a_{\kappa\kappa}|\leqq P_\kappa$. This result will be improved as follows. Each characteristic root must lie in at least one of the $C_{n,2}$ ovals of Cassini $|z-a_{\kappa\kappa}||z-a_{\lambda\lambda}|\leqq P_\kappa P_\lambda$. It follows in particular that, for $\nu=1, 2, \cdots, n$, $|\omega_\nu|\leqq\max_{\kappa,\lambda=1,2,\cdots,n}\{|a_{\kappa\kappa}|+(P_\kappa P_\lambda)^{1/2}\}$. Received September 25, 1946.)

348. H. W. Brinkmann: *On the prime divisors of a polynomial with integral coefficients.*

Let $f(x)$ and $g(x)$ be polynomials with rational integral coefficients. It is shown in this paper that there exist infinitely many primes p that are divisors of both polynomials, that is, there exist integers a, b so that $f(a)\equiv g(b)\equiv 0 \pmod{p}$. The proof is not elementary. As a corollary it follows that it is impossible for all but a finite number of prime divisors of a polynomial to belong to the same residue class $kz+m$ if $m\not\equiv 1 \pmod{k}$. This answers a question raised by A. T. Brauer (Duke Math. J. vol. 13 (1946) pp. 235–238). The problem of deciding when there are infinitely many prime divisors of $f(x)$ that are *not* divisors of $g(x)$ is also attacked and as a consequence it is possible to generalize Brauer's main theorem in the paper just referred to. (Received August 26, 1946.)

349. E. R. Kolchin: *Algebraic matric groups and the Picard-Vessiot theory of homogeneous linear ordinary differential equations.*

This is a modern algebraic development of the Galois theory of homogeneous linear ordinary differential equations of Picard and Vessiot. Based on a theory of algebraic groups of matrices (developed here, for any algebraically closed field of arbitrary characteristic, without recourse to Lie theory) and on the Ritt theory of algebraic differential equations, the paper extends the Picard-Vessiot theory, fills in its main gaps, and brings it up to present day standards of rigor. The coefficient domain is an arbitrary differential field of characteristic 0 with an algebraically closed field of constants. Among the results obtained are the analogue to the fundamental theorem of Galois theory (one-to-one correspondence between subgroups and intermediate fields), and an extension of Vessiot's big theorem on solvability "by quadratures." The first part of the development is quite general and contains the rudiments of a general Galois theory of differential fields (ordinary or partial) of characteristic 0. (Received September 16, 1946.)

350. E. R. Kolchin: *Extensions of differential fields.* III.

The purpose of the present note is to show how the point of view of a preceding paper (*Extensions of differential fields.* I, Ann. of Math. vol. 43 (1942) pp. 724–729) can be used in developing the concepts of resolvent, dimension and order introduced by J. F. Ritt in his theory of algebraic differential equations. (Received September 16, 1946.)

351. R. M. Robinson: *Unsymmetrical approximation of irrational numbers.*

As an application of a theorem about lattice points, B. Segre (Duke Math. J. vol. 12 (1945) pp. 337–365) has shown that for any $\tau \geqq 0$ every irrational number ξ admits infinitely many rational approximations A/B such that $-1/(1+4\tau)^{1/2}B^2 < A/B-\xi<\tau/(1+4\tau)^{1/2}B^2$. For $\tau=1$, this reduces to the inequality $-1/5^{1/2}B^2 < A/B-\xi<1/5^{1/2}B^2$ of Hurwitz. For other values of τ, one side of the inequality is strengthened, the other weakened. In this paper, a proof of Segre's theorem is given using continued fractions. Some sharper results are also obtained; in particular, it is shown that for $\epsilon>0$ the inequality $-1/(5^{1/2}-\epsilon)B^2<A/B-\xi<1/(5^{1/2}+1)B^2$ has infinitely many solutions. This result is interesting since it shows that one side of Hurwitz's inequality can be strengthened without essentially weakening the other. (Received September 20, 1946.)

352. A. R. Schweitzer: *Sums and products of ordered dyads in the foundations of algebra.* V.

Postulates for a group of ordered dyads are constructed in terms of elements of a set $S(\alpha)$ and a relation of equality (equivalence) between dyads $T(\alpha\beta)$ of these elements, as follows: 1. α, β in S imply $\alpha\beta$ in T, and conversely. 2. $\alpha\alpha$, $\beta\beta$ in T imply $\alpha\alpha=\beta\beta$. 3. $\alpha\beta$, $\gamma\delta$ in T imply the existence of ξ, η in S such that $\beta\xi=\gamma\delta$ and $\eta\gamma=\alpha\beta$. 4. $\alpha\beta=\alpha\xi$ implies $\xi=\beta$ and $\alpha\beta=\eta\beta$ implies $\eta=\alpha$. Definitions. 1. $\alpha\gamma$ is the "product" of $\alpha\beta$ and $\beta\gamma$. 2. $\lambda\mu$ is the product of $\alpha\beta$ and $\gamma\delta$, $\alpha\beta\times\gamma\delta=\lambda\mu$, means: There exists ξ such that $\beta\xi=\gamma\delta$ and $\alpha\xi=\lambda\mu$ or there exists η such that $\alpha\beta=\eta\gamma$ and $\eta\delta=\lambda\mu$. Relatively to the preceding postulates and definitions, any complete set of mutually nonequivalent dyads constitutes a group with a self-conjugate dyad $(\alpha\alpha)$ as the identical element. Examples: (1) Any regular group of configurational sets of dyads: two dyads are equivalent if and only if they occur in the same configurational set. (2) The set of segments of points on a descriptive line (including null segments): $\alpha\beta=\gamma\delta$ if and only if $\alpha\beta$ and $\gamma\delta$ are congruent and have the same orientation. In the latter example $\alpha\gamma$ is usually designated as the "sum" of $\alpha\beta$ and $\beta\gamma$. (Received September 27, 1946.)

353. A. R. Schweitzer: *Sums and products of ordered dyads in the foundations of algebra.* VI.

The construction of postulates for a field of dyads is based on set S of elements (α) and a set T of ordered dyads (α/β): A_1. There exists ω uniquely in S such that for any α in S, α/ω is not in T. A_2. α, β in S, $\beta\neq\omega$, imply α/β in T and conversely. A_3. ω/ξ, ω/η in T, $\xi\neq\eta$, imply $\omega/\xi=\omega/\eta$. B_1. Excluding ω/ξ, the dyads of the set T form a group under multiplication relatively to the postulates and definitions of the preceding abstract (with $\alpha\beta$ replaced by α/β). B_3. Multiplication is commutative. C. The dyads of T, including ω/ξ, form a commutative group under the undefined compostion "addition." Under C postulates are first stated (C_1) in terms of addition of dyads with the same posterior elements (denominators); it is then assumed (C_2) that α/β, γ/δ in T imply the existence of ξ such that $\gamma/\delta=\xi/\beta$. Definition: $\alpha/\beta+\gamma/\delta=\lambda/\mu$ means: There exist ξ, η such that $\gamma/\delta=\xi/\beta$, $\eta/\beta=\lambda/\mu$, where $\alpha/\beta+\xi/\beta=\eta/\beta$. D. Multiplication over addition to the right (left) is distributive. The independence of C_2 is discussed. Application is made to the author's *Foundations of Grassmann's extensive algebra* (Amer. J. Math. vol. 35 (1913) pp. 39–49). (Received September 27, 1946.)

354. J. D. Swift: *Periodic functions over a finite field.*

A function of period a over the Galois field, $GF(p^n)$, is defined as a function over the elements of the field to the elements of the field such that $f(x+na)=f(x)$, where n is an integer. Multiply periodic functions are defined in an analogous manner. A $GF(p^n)$ admits functions with a number of independent periods not greater than $n-1$. The basic function of period a is: $f(a;x)=x^p-a^{p-1}x$. The basic function of periods $a_1, \cdots, a_n$ may be defined recursively from the above as: $f(f(a_1, \cdots, a_{n-1}; a_n); f(a_1, \cdots, a_{n-1}; x))$. These basic functions are odd and additive, and $f(a_1, \cdots, a_n; cx)$ is periodic with periods $a_1/c, \cdots, a_n/c$. The principal result is: *Any periodic function of periods $a_1, \cdots, a_k$ over the $GF(p^n)$, $k\leqq n-1$, may be expressed by:* $g(x)=\sum_1^l\alpha_if^i(a_1, \cdots, ak; x)+\alpha_0)$, where $l=p^{n-k}-1$, *and the α_i are a set of elements of the field.* (Received September 16, 1946.)

355. P. M. Whitman: *Finite groups with a cyclic group as lattice-homomorph.*

It is shown that if G and H are groups, $L(G)$ and $L(H)$ their lattices of subgroups, $L(G)$ is finite, $L(H)$ is a lattice-homomorphic image of $L(G)$, and H is cyclic, then G contains a cyclic subgroup which is mapped onto H by the homomorphism. (Received September 26, 1946.)

Analysis

356. Warren Ambrose: *Direct sum theorem for Haar measures.*

A variation of a theorem of A. Weil (*L'intégration dans les groupes topologiques*, Paris, 1940, pp. 42–45) is proved. (Received September 19, 1946.)

357. R. F. Arens: *Location of spectra in Banach $*$-algebras.*

A Banach $*$-algebra A is a Banach space with a continuous multiplication and a $*$-operation satisfying $(\lambda f+\mu g)^*=\bar{\lambda}f^*+\bar{\mu}g^*$, $f^{**}=f$, $(fg)^*=g^*f^*$, and $k|f|\,|f^*|\leqq|ff^*|$, $k>0$. One can renorm A such that $\|fg\|\leqq\|f\|\,\|g\|$, and there will exist $k'>0$ such that $k'\|f\|\,\|f^*\|\leqq\|ff^*\|$. It is proved that, if $f=u+iv$, $u^*=u$, $v^*=v$, $uv=vu$, then $|x\cos\theta+y\sin\theta|\leqq\|u\cos\theta+v\sin\theta\|$ for any complex number $x+iy$ in the spectrum of f, and $0\leqq\theta\leqq 2\pi$. Thus if $f=f^*$, the spectrum is real. The case $k'=1$ studied by I. Gelfand and M. Neumark, Rec. Math. (Mat. Sbornik) N. S. vol. 12 (1943) pp. 197–213, is a special case. (Received August 8, 1946.)

358. Lipman Bers: *A property of bounded analytic functions.*

Let $f(z)$ be bounded and analytic for $|z|<1$. If $\{z_n\}$ is a sequence of points such