25. S. E. Warschawski: *On the modulus of continuity of the mapping function at the boundary in conformal mapping.*

The author proves the following theorem: Let D be a simply connected region such that (i) D contains the unit circle and is contained in $|w|<R$; (ii) if D is divided into two parts by a crosscut of diameter $\delta<1$, then the diameter d of the subregion of D which does not contain the origin satisfies the inequality $d\leqq\mu\delta+\eta$ (μ and η are constants, $\mu\geqq 1$, $\eta\geqq 0$). Suppose that $w=f(z)$ maps $|z|<1$ conformally onto D ($f(0)=0, f'(0)>0$). Let k be an arbitrary constant, $k\geqq 4R^2$, $k>\mu^2$. If z_0 is any point on $|z|=1$ and if z_1 and z_2 are in $|z|<1$ with $|z_i-z_0|\leqq r\leqq\exp[-k\pi^2/4]$, then $|f(z_1)-f(z_2)|\leqq(kr^\alpha/\mu)+\eta k^{1/2}/(k^{1/2}-\mu)$ where $\alpha=(2/(\pi^2k))(\log k-2\log\mu)$. This result is applied to the following problem. Let C_1 and C_2 be closed Jordan curves containing $w=0$ in their interiors D_i, such that C_2 is in the ϵ-neighborhood of C_1 (that is, every point of C_2 is within a circle of radius ϵ about some point of C_1) and C_2 in the ϵ-neighborhood of C_1. Let $w=f_i(z)$ map $|z|<1$ onto D_i ($f_i(0)=0, f_i'(0)>0$). Then a function $\Phi(\epsilon)$ is determined, which, aside from ϵ, depends only on certain parameters characterizing the C_i, such that $|f_1(z)-f_2(z)|\leqq\Phi(\epsilon)$ for $|z|\leqq 1$. (Received October 19, 1945.)

26. J. W. T. Youngs: *Various definitions of surface and area.* Preliminary report.

This paper lists several definitions of the word "surface" and uses recent results in the field to show how the term "area" can be applied to each. The principal result is that in each case the area is a lower semi-continuous function of the surface. (Received October 17, 1945.)

Applied Mathematics

27. Edward Kasner and John DeCicco: *Heat surfaces.*

If a region of space is heated by conduction, the temperature ν at a time t at a point (x, y, z) is $\nu=\phi(x, y, z, t)$, where ϕ satisfies the Fourier heat equation. Kasner has introduced the term heat surfaces for those along which $\nu=$const. and $t=$const. In general, there are ∞^2 heat surfaces. In the present work, the authors extend to space certain theorems of Kasner concerning heat families in the plane, published in 1932–1933, Proc. Nat. Acad. Sci. U.S.A. There are no systems of ∞^2 planes or ∞^2 spheres which form a heat family except in the imaginary domain. The only sets of ∞^1 planes which form a heat family are the pencils. A system of ∞^1 spheres is a heat family if and only if it is a concentric set. The only isothermal systems of planes are pencils and the only isothermal sets of spheres are concentric families. The cases where there are only ∞^1 heat surfaces are connected with the equations of Laplace, Poisson, and Helmholtz-Pockels. Finally, these results are extended to n dimensions. (Received October 11, 1945.)

28. H. E. Salzer: *Coefficients for repeated integration with central differences.*

The present paper is aimed toward facilitating double or k-fold repeated quadrature of a function which is tabulated at a uniform interval, with its central differences of even order (see Abstract 51-9-172). When the Everett interpolation formula is integrated k times over an interval of tabulation, one obtains a formula for stepwise multiple quadrature in the form (1) $\int_{x_0}^{x_1}\cdots\int_{x_0}^{x}\int_{x_0}^{x}f(x)(dx)^k=h^k[A_0^{(k)}f_0+B_0^{(k)}f_1$

$+\sum_{s=1}^{m}(A_{2s}^{(k)}\delta_0^{2s}+B_{2s}^{(k)}\delta_1^{2s})]+R_{2m}$. This article tabulates: I. $A_{2s}^{(2)}$, $B_{2s}^{(2)}$, exact values for $2s=0, 2, \cdots, 20$ and sixteen decimal places for $2s=22, \cdots, 48$. II. $A_{2s}^{(k)}$, $B_{2s}^{(k)}$, $k=3(1)6$, $2s=2, 4, \cdots, 20$, eight significant figures (but exactly for $s=0$). Simple recursion formulas are obtained for $A_{2s}^{(2)}$ and $B_{2s}^{(2)}$ in terms of $M_{2s}\equiv 2A_{2s}^{(1)}$, an application of which is the expression of (1) in terms of δ_0^{2s} and $\delta_{1/2}^{2s+1}$, analogous to the forward version of the Newton-Gauss interpolation formula. Expressions are derived for $A_{2s}^{(2)}$ and $B_{2s}^{(2)}$ in terms of $B_\nu^{(n)}(x)$, Bernoulli polynomials of degree ν and order n. Cumulative recursion formulas are derived for $A_{2s}^{(k)}$ in terms of $B_{2s}^{(i)}$, and for $B_{2s}^{(k)}$ in terms of $A_{2s}^{(i)}$, $i=1, 2, \cdots, k$. (Received November 22, 1945.)

29. H. E. Salzer: *Table of coefficients for obtaining the first derivative without differences.*

When a function $f(x)$ is known for n equally spaced arguments at interval h, an approximation to the derivative at a point $x=x_0+ph$ may be obtained, by the differentiation of the well known Lagrangian interpolation formula, in the form $f'(x_0+ph)\sim(1/hC(n))\sum_{i=-[(n-1)/2]}^{i=[n/2]}C_i^{(n)}(p)f(x_0+ih)$, where $[m]$ denotes the largest integer in m, $C_i^{(n)}(p)$ are polynomials in p of the $(n-2)$th degree, and $C(n)$ denotes the least positive integer which enables $C_i^{(n)}(p)$ to have integral coefficients. The present table gives the exact values of these polynomials $C_i^{(n)}(p)$, for p ranging from $-[(n-1)/2]$ to $[n/2]$. For $n=4$, 5 and 6, the polynomials $C_i^{(n)}(p)$ are tabulated at intervals of 0.01; for $n=7$, they are tabulated at intervals of 0.1. (Received November 6, 1945.)

30. A. C. Sugar: *On the numerical treatment of forced oscillations.*

The solution of the equation $\ddot{x}+\omega^2x=a(t)$, $x(0)=0=\dot{x}(0)$, is given by $x=(1/\omega)\cdot\int_0^t a(\tau)\sin\omega(t-\tau)d\tau$. In this paper a simple approximation of x and hence of $\ddot{x}$ is found. Easy vector methods of obtaining max $|x|$ and max $|\ddot{x}|$ are discussed. (Received November 17, 1945.)

Geometry

31. L. M. Blumenthal: *Metric characterization of elliptic space.*

In this paper the first characterizations of finite and infinite dimensional elliptic spaces to be expressed wholly and explicitly in terms of distance relations are obtained. The characterizations are secured by direct, elementary geometric arguments. Only the simplest properties of elliptic space are used and no reference whatever is made to topological theorems. (Received October 3, 1945.)

32. S. C. Chang: *A new foundation of the projective differential theory of curves in five-dimensional space.*

As a preliminary a covariant triangle of reference and unit point for a plane curve is determined in an elementary and geometric manner using neighborhoods of order six. For a point P on a curve Γ in five dimensions a covariant triangle PP_1P_2 and unit point is first determined for the curve of intersection C of osculating plane and developable hypersurface of Γ. PP_1P_2 are three vertices of a quadrilateral Q on a covariant quadric generated by certain Bompiani osculants. The fourth vertex of Q is chosen as P_3. Similarly P_4, P_5 can be defined leading to a covariant pyramid for Γ. The Frenet-Serret formulas for the cases of P an ordinary and a k-ic ($k=6, 7, 8$) point follow from the corresponding canonical expansions. The method has the ad-