Lectures on the theory of functions. By J. E. Littlewood. Oxford University Press, 1944. 8+244 pp. 17s 6d.

This book is neither a systematic treatise on the theory of functions nor a monograph on some definite branch of this theory, but rather it considers such parts of the theory of functions as are near to the center of the sphere of interest of the author. The book has a strong and unmistakable personal character and can give not only to those who know the author personally but also to those who read between the lines an impression of his power and penetration. Mathematical arguments are selected and arranged with exceptional regard for methodological points, an approach which is extremely instructive and valuable.

The book is divided into three parts: an extensive "Introduction" which occupies more than one-third of the book, Chapter I and Chapter II. According to the author's original plan, further chapters were to be incorporated in a second volume which has not yet been published. The various matters collected for reference in the Introduction provide a mathematical background for the understanding of the book, but much of the preparation is actually intended for the unpublished second volume.

The Introduction contains a systematic treatment of the inequalities of Hölder and Minkowski. A section on the theory of functions of a real variable provides the background needed in that field of the theory of functions of a complex variable which concerns "boundary values." Another section is devoted to the general theory of harmonic functions, and a section at the end sets out the behavior of certain special functions whose role is to provide counter examples.

Chapter I is composed of selected topics from classical theory of functions of a complex variable, and ends with a discussion of the theory of conformal mapping.

Chapter II is centered around the following general problem. Suppose that some restriction is placed on the set of values taken by a function $f(z)$ which is regular in the unit circle $|z|<1$. What is the influence of this restriction on the behavior of the function? In particular, how does it restrict the modulus of the function and its coefficients? For example, the point $w=f(z)$ may be restricted to move, for varying z, on some given Riemann surface (subordination). The chapter begins with a section on subharmonic functions (a topic not prima facie related to the main problem), and proceeds to a discussion of the principle of subordination (Lindelöf principle). Applications of this principle are made to functions subordinate to the elliptic modular functions, and also to functions subordinate to schlicht functions.

Most of the chapter is based on the author's own research, but some of the material has been supplied by W. W. Rogosinski, who helped the author in the final writing of this chapter.

Some readers will miss an index, and many more will miss bibliographical references.

D. C. SPENCER

General equilibrium theory in international trade. By Jacob L. Mosak. (Cowles Commission Monograph No. 7.) Bloomington, Ind., Principia Press, 1944. 187 pp. $2.50.

This work can be recommended to mathematicians who are interested in learning what modern economic theory is about or what mathematical methods have been found useful in this field. It is an interesting historical fact that a statistically significant fraction of the great literary or non-mathematical economists began their training as mathematicians. On the other hand, a no less significant proportion of those who have made important contributions to mathematical economics started out with poor early trainings in mathematics. And with a few notable exceptions, the excursions into economics of well-trained creative mathematicians of reputations have not resulted in the most important advances in this field.

Like Willard Gibb's work in classical thermodynamics, but unlike his statistical mechanics, the tools employed in this book are the mathematically elementary ones capable of being taught in advanced calculus and undergraduate algebra classes: maxima of functions of many variables, Jacobians, Hessians, determinants, and so on. However, some of the important aspects of these topics are rarely taught in such courses although there is no good reason why they should not be. Among these is the statement of sufficient secondary conditions for an extremum of a function of many variables subject to a number of constraints; or what is the same thing, the conditions for definiteness of a quadratic form of variables subject to linear constraints, with the implied inequalities of certain bordered Hessians. The Weierstrassian treatment of this problem is not widely available. (Compare, however, Harris Hancock's *Maxima and minima*, now out of print, and Carathéodory's *Partiale Differentialgleichungen erster Ordnung*.) Because Pareto was unfamiliar with this theory, progress was held up for fifteen years until the Russian mathematician Slutsky put the cart back on the tracks.

In connection with asymmetrical determinants which arise in the study of the problem of stability of multiple markets, some less familiar mathematical problems and theorems occur. Typical of these is