

theoretical mathematical statistician and the run-of-the-mine working statistician.

As the book stands, it fits the more important findings in the field of advanced mathematical statistical theory into their proper places in a unified treatment providing uniform notation and terminology. Such a treatment will clearly prove of great value to college and university students, and also will facilitate important technical statistical work.

The reviewer wishes to acknowledge gratefully the assistance of Dr. Bradford F. Kimball of the New York State Public Service Commission. It is to be noted, however, that Dr. Kimball does not necessarily agree with all of the conclusions.

R. W. BURGESS

*A treatise on the theory of Bessel functions.* By G. N. Watson. 2d ed. Cambridge University Press; New York, Macmillan, 1944. 6+804 pp. \$15.00.

The first edition of this book was published in 1922 and was reviewed by Professor R. D. Carmichael (Bull. Amer. Math. Soc. vol. 30 (1924) pp. 362-364). At that time the book constituted an almost encyclopedic account of results concerning Bessel functions which had been obtained by various mathematicians during the previous century. Furthermore, it was written in such a manner as to be comprehensible for those readers in possession of the fundamentals of modern analysis.

The present edition makes no attempt to take account of additions to the literature during the period since 1922, and the changes are for the most part limited to correction of minor errors. However, the book still remains a very comprehensive account of the bulk of the literature on the field in question. It will undoubtedly be of great service to workers in pure and applied mathematics who have occasion to make use of the various formulas and theories connected with Bessel functions.

There exists, however, one rather curious gap in the original edition which has not been filled in the current one. The original investigation by Fourier of a problem concerning the flow of heat in a cylinder, which led to the development of an arbitrary function in a series of Bessel functions of order zero, is outlined in §1.5. Reference is there made to Chapter 18 for a discussion of the validity of the expansion. But in order to show that the solution which Fourier gave for his problem is really valid, it is necessary to establish the uniform

convergence or the uniform summability of the development in the neighborhood of the origin. In Professor Watson's book the uniform convergence of the development in this neighborhood after multiplication by the factor  $x^{1/2}$  is established, but since this factor vanishes at the origin, this result throws no light on the uniform convergence of the development itself in the neighborhood in question.

As far as the reviewer is aware, the only proofs in the literature that Fourier's solution is mathematically valid are to be found in three papers published by him in 1909, 1911, and 1920 (Trans. Amer. Math. Soc. vol. 10 (1909) pp. 391-435; vol. 12 (1911) pp. 181-206; vol. 21 (1920) pp. 107-156). The second paper furnishes a proof based on uniform convergence; the first and third combined furnish a proof based on uniform summability. Professor Watson refers to these papers in his Chapter 18 but fails to point out that results of this sort are essential as a justification of Fourier's solution.

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