ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

51. E. W. Barankin: *Bounds for the characteristic roots of a matrix.*

With the application of Schwarz's inequality (a principal tool in the more recent investigations of bounds for matrix roots) to the n linear equations $\lambda x_r = \sum_s a_{rs}x_s$, where λ is a root of the arbitrary matrix $A = \|a_{rs}\|$, and $\{x_r\}$ is an eigen-vector of A belonging to λ, it is proved that $|\lambda| \leqq \max (R_r T_r)$, where $R_r \equiv \sum_s |a_{rs}|$, $T_r \equiv \sum_s |a_{sr}|$. Generalizations of this theorem, to give stronger upper bounds, are indicated; and the independent result, that $|\lambda| \leqq \max R_r$, $|\lambda| \leqq \max T_r$, is established. The first theorem was recently proved for matrices of order 2 by A. B. Farnell (Bull. Amer. Math. Soc. vol. 50 (1944) pp. 789–794). (Received December 23, 1944.)

52. Garrett Birkhoff: *Lattice-ordered Lie groups.*

A Lie l-group is defined as a lattice-ordered group which is a Lie group; a Lie tl-group, as a Lie l-group whose lattice operations are continuous in the topology; a Lie l-algebra, as a lattice-ordered Lie algebra whose set of positive elements is invariant under all inner automorphisms. Conditions that the Lie algebra of a Lie group be a Lie l-algebra are found. A Lie algebra of dimension r can be made into a Lie l-algebra if and only if it has a chain of invariant subalgebras of length r; hence only if it is solvable. A simply ordered topological l-group is one-dimensional. The only Lie tl-groups are the powers R^n of the additive group of real numbers; hence they are all commutative. (Received January 3, 1945.)

53. A. L. Foster: *Boolean-like rings, a generalization of Boolean rings. The logical algebra of general commutative rings.*

This paper is mainly concerned with the study of a generalization (first touched on in: A. L. Foster, *The idempotent elements of a commutative ring form a Boolean algebra* · · · . *Ring duality and transformation theory*, to appear in Duke Math. J., March 1945) of the concept Boolean ring, a generalization in which many of the formal properties, both ring and "logical," of the latter are preserved, and one which arises naturally from the basic duality theory of rings previously introduced. The class of Boolean-like rings is discussed within the framework of a certain logical algebra of (general) rings. (Received January 8, 1945.)

54. G. N. Garrison: *Note on invariant complexes of a quasigroup.*

In an earlier paper (*Quasi-groups*, Ann. of Math. vol. 41 (1940) pp. 474–487) the writer discussed (§4.4) relations between two invariant complexes, H and K, of a finite quasigroup S. The purpose of the present note is to discuss the intersection $H \wedge K$ under much less restrictive hypotheses than were imposed earlier. If $H \wedge K = D$, D not vacuous, D is shown to be invariant. Also the order of D divides the order of H (and K) and non-overlapping sets $D, Da, Db, \cdots$ exist such that $H = D + Da + Db + \cdots$. When K is invariant, so is Ka for any a (previous result); therefore if $H \wedge K = 0$, choose $a < S$ so that $H \wedge Ka = D \neq 0$, D invariant. In either case it is shown that, whenever $Hb \wedge Kc = D \neq 0$, D is invariant and any other intersection $Hp \wedge Kq$ is either vacuous or takes the form Dr for some r. (Received January 25, 1945.)

55. R. E. Johnson: *Systems of finite linear equations.*

In this paper the author studies the system of linear equations (1) $\sum_{r=1}^{n_s} a_{rs}x_r = y_s$, $s = 1, 2, \cdots$ for a_{rs} and y_s in a commutative field P. It is shown that the matrix $A = (a_{rs})$ has associated with it a "quasi-inverse" A^*. A necessary and sufficient condition that (1) have a solution is that the idempotent matrix A^*A be a right unit of the matrix $Y = (y_{rs})$; $y_{1s} = y_s$, $y_{rs} = 0$, $r > 1$. If (1) has a solution, the general solution is obtained in terms of A and A^*. (Received January 23, 1945.)

56. J. N. B. Livingood: *A partition function with the prime modulus $p > 3$.*

In this paper the author, generalizing a problem recently solved by J. Lehner, obtains a convergent series for $p_a(n)$, the number of partitions of n into summands which are congruent to $\pm a$ modulo p, p being a prime number greater than 3. The method is that of Hardy and Ramanujan in its recent refinement. Let $p_a(n)$ be the coefficient of a generating function $F_a(x)$ which is regular within the unit circle. All depends upon the asymptotic behavior of $F_a(x)$ near roots of unity. In order to determine this a modular transformation is applied to $F_a(x)$. The infinite product $F_a(x)$ goes over into another one whose asymptotic behavior is evident. The coefficients $p_a(n)$ are then computed by means of the Cauchy integral, subject to the Farey dissection. Since the modular forms in question are of dimension zero, the method, due to Rademacher, necessitates the introduction of the so-called Kloosterman sums, in order to lower the estimate of the sums of certain roots of unity. An interesting by-product of the investigation is the determination of the asymptotic ratios of the $p_a(n)$ for different a and for n tending to infinity. (Received January 5, 1945.)

57. M. F. Smiley: *A remark on metric boolean rings.*

Just as a metric lattice is modular (V. Glivenko, *Géométrie des systèmes de choses normées*, Amer. J. Math. vol. 58 (1936) pp. 799–828), so is a ring with unity element on which is defined a real valued function $\mu(a)$ satisfying (1) $\mu(a) > 0$ *for every* $a \neq 0$ and (2) $\mu(a+b) + 2\mu(ab) = \mu(a) + \mu(b)$ a metric boolean ring. An example shows that the assumption of a unity element is essential. (Received January 16, 1945.)

58. Ernst Snapper: *Partial differentiation and elementary divisors.*

Let $P = K[x_1, \cdots, x_n]$ be a polynomial ring, where K is a field of characteristic p $(=0$ or $\neq 0)$. The operator ring $D = K[\partial/\partial x_1, \cdots, \partial/\partial x_n]$ and the "algebra of linear exponentials M" are defined. Elements of D are operators $\phi = \sum_{i_1 \cdots i_n} a_{i_1 \cdots i_n} \partial^{i_1 + \cdots + i_n}/\partial x_1^{i_1} \cdots \partial x_n^{i_n}$, operating on the elements of M. A set of linear partial differential equations with constant coefficients is a set S of operators and its solutions are elements $\mu \in M$, where $\phi(\mu)=0$ for all $\phi \in S$. An "auxiliary" ideal $\mathfrak{a} \subseteq P$ corresponds to S. The general point ξ of an $(n-i)$-dimensional associated prime $\mathfrak{p}$ of a primary component $\mathfrak{q}$ of $\mathfrak{a}$ determines a "general" $(n-i)$-dimensional exponential solution exp $(\xi) \in M$ of S. The differential exponent of $\mathfrak{p}$ is δ if a polynomial f with i variables and with coefficients in the field of rational functions of $\mathfrak{p}$, where f exp(ξ) is a solution of S, has at most degree $\delta-1$. The relationships are studied between δ, the ordinary exponent ρ of $\mathfrak{q}$, the Hentzelt exponent ν of $\mathfrak{q}$, and the multiplicity σ of the root which corresponds to $\mathfrak{p}$ of the $(n-i)$-dimensional elementary divisor of $\mathfrak{a}$. When $\mathfrak{p}$ is isolated and either $p=0$ or $\delta<p$, $\rho=\nu=\sigma=\delta$. When $p\neq 0$, and $\delta \geqq p$, $\rho \leqq \nu$, $p \leqq \nu$, $p \leqq \sigma$. (Received January 17, 1945.)

59. W. J. Sternberg: *On a special set of algebraic nonlinear equations.*

Some physical problems lead to algebraic nonlinear equations, such as the following set $\sum_{i=1}^{3} 1/(x_i/s_k+jy_i)=1/(a_k/s_k+jb_k)$, where $k=1, 2, 3$, $j=(-1)^{1/2}$, $s_1^2\neq s_2^2$, $s_2^2\neq s_3^2$, $s_3^2\neq s_1^2$. An analogous set of two or four or more equations could also be treated. The unknowns are x_i, y_i, while a_k, b_k, s_k are given. The above complex equations are equivalent to six real equations. The transformation $u_i=y_i/x_i$, using the abbreviation $1/(1+s_k^2u_i^2)=f_k(u_i)$ $[i, k=1, 2, 3]$, leads to $\sum_{i=1}^{3} f_k(u_i)/x_i=A_k$, $\sum_{i=1}^{3} u_if_k(u_i)/x_i=B_k$ where A_k, B_k can be computed from a_k, b_k, s_k. Since the above equations are linear with respect to $1/x_i$, eliminate these unknowns and obtain for the u_i three nonlinear equations, whose left sides are determinants. These determinants are rational alternating functions of the u_i. Simplify the said equations and introduce the elementary symmetric functions w_1, w_2, w_3 of the u_i. The point is that finally three linear equations are obtained for w_1, w_2, w_3. They are uniquely determined and the equation with the roots u_1, u_2, u_3 can be found. The problem is therefore reduced to one equation of third degree. (Received January 20, 1945.)

60. H. S. Wall: *Polynomials with real coefficients whose zeros have negative real parts.*

Let $P(z)=z^n+a_1z^{n-1}+a_2z^{n-2}+\cdots+a_n$ be a polynomial with real coefficients, and let $Q(z)$ be the polynomial obtained from $P(z)$ by dropping out the first, third, fifth, $\cdots$ terms of $P(z)$. Then, the zeros of $P(z)$ all have negative real parts if and only if the successive quotients obtained in applying to $P(z)$ and $Q(z)$ the euclidean algorithm for finding the greatest common divisor of two polynomials have the form c_1z+1, c_2z, c_3z, $\cdots$, c_nz, where c_1, c_2, $\cdots$, c_n are positive. On the basis of this result, a new proof is obtained of a theorem of A. Hurwitz (*Werke*, vol. II, p. 533 ff.). (Received January 24, 1945.)

Analysis

61. Felix Bernstein: *The integral equations of the theta function.*

In 1920 the author showed that the integral equation $\vartheta*\vartheta-2t\vartheta+\vartheta*1-1=0$ $(\vartheta*\vartheta=\int_0^t\vartheta(\tau)\vartheta(t-\tau)dt)$ for the variable $h=e^{-\pi^2t}$ has the theta zero function $\sum_{-\infty}^{+\infty}h^{n^2}$ as the only solution analytical and regular in the unit circle at the origin. In subsequent papers it has been brought out that this equation defines relationships of the theta zero function to the theory of heat and a number of new theorems. The transcendent theorems of addition have been derived. It has been shown that the Volterra theory