ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

198. Warren Ambrose: *Structure theorems for a special class of normed rings.*

Normed rings (in the sense of Gelfand, but without assumptions of commutativity or of a unit) are considered which satisfy the following two conditions: (1) the underlying Banach space is a Hilbert space, (2) to each x in the ring there is an "adjoint," x^*, with the properties that the linear transformations defined by $y \rightarrow x^*y$, and $y \rightarrow yx^*$ are the adjoint operators of $y \rightarrow xy$ and $y \rightarrow yx$. The structure of such rings is completely determined. It is shown that such a ring is always a direct sum of simple rings and that a simple ring is always a full matrix ring, of either finite or infinite dimension. By a full matrix ring of infinite dimension is meant here the set of all functions of two variables (complex-valued) for which the sum of the squares of the absolute values of the elements converges, with the expected definitions of the operations in the ring. (Received July 6, 1944.)

199. E. T. Bell: *Separable diophantine equations.*

A monomial in the independent indeterminates $x_1, \cdots, x_n$ is called elementary if at least one of the indeterminates occurs only to the first power. A sum of elementary monomials with integer coefficients, equated to zero, is called an extended multiplicative equation. Systems of extended equations are called separable if their complete integer solution is reducible to that of a multiplicative system (methods for the complete solution of which are known) and a system of linear diophantine equations, or to either of these alone. Among the applications are the complete integer solution of $S_m = S_n$, where S_m is a sum of m squares with arbitrary integer coefficients. The methods used apply to diophantine equations in any unique factorization domain, in particular to an integral domain with a euclidean algorithm. The paper will appear in the Transactions. (Received July 10, 1944.)

200. L. M. Blumenthal: *Note concerning an extension of the notion of matrix rank.*

Investigations concerning the imbedding of metric spaces in elliptic spaces suggested an extension of the notion of matrix rank in terms of which certain metric properties of the space could be algebraically formulated and described in a better way than by the use of classical concepts. If $\mathcal{A} = (a_{ij})$, $\mathcal{E} = (\epsilon_{ij})$ are two $m \times n$ matrices, $\mathcal{A}$ is said to have $\mathcal{E}$-relative rank r provided the matrix $(a_{ij}\epsilon_{ij})$ $(i = 1, 2, \cdots, m;$

$j=1, 2, \cdots, n$) has rank r. The applications of matrix-relative rank so far deal only with the case in which $\mathcal{A}$ and $\mathcal{E}$ are symmetric (square) matrices, with $\epsilon_{ii}=1$, $\epsilon_{ij}=\epsilon_{ji}=\pm 1$. A fundamental problem is that of extending to matrix-relative rank the theorem of Frobenius connecting the ordinary rank of a symmetric matrix with the ranks of certain of its principal minors. Among the theorems of this note, it is shown that the symmetric matrix $\mathcal{A}=(a_{ij})$, $a_{ii}=1$, $0 \leqq a_{ij} < 1$, $i \neq j$ $(i, j=1, 2, \cdots, m)$, has an $\mathcal{E}$-relative rank 2 provided each fourth-order principal minor has an $\mathcal{E}$-relative rank 2 (where $\mathcal{E}$ represents generically any of the class $\{\mathcal{E}\}$ of symmetric matrices with $\epsilon_{ii}=1$, $\epsilon_{ij}=\pm 1$), while if each $a_{ij}>0$ and $n>4$, then $\mathcal{A}$ has an $\mathcal{E}$-relative rank 2 whenever each *third-order* principal minor has, *unless every element of* $\mathcal{A}$ *is either* $1/2$ *or* $3^{1/2}/2$. (Received July 6, 1944.)

201. H. W. Eves: *Some associated theorems on matrices and determinants.*

In offering a vector-matrix treatment of projective geometry it was found that certain portions of the subject can be elegantly handled with the use of a small group of associated theorems concerning vectors, matrices, and determinants. These theorems are interesting in themselves and are presented here isolated from their geometrical application. (Received June 10, 1944.)

202. Nathan Jacobson: *Structure theory of simple rings without finiteness assumptions.*

In the first part of the paper the structure is obtained of arbitrary simple associative rings that either (1) contain a minimal right ideal or (2) contain a maximal right ideal. Representations of these rings are found as certain types of rings of linear transformations in vector spaces over division rings. The question of the invariance of the representation is discussed in the case (1). In the second part of the paper simple algebras, including simple non-associative algebras, with a possibly infinite basis are considered. A definition of the extended center and of central simple algebra for the infinite dimensional case is given. It is proved that a central simple algebra remains simple when the underlying field is arbitrarily extended. (Received June 9, 1944.)

203. B. W. Jones: *A canonical quadratic form for the ring of 2-adic integers.*

Types of transformations are given by which any form in the ring of 2-adic integers may be reduced to a unique canonical form. This establishes a means of ascertaining the equivalence or non-equivalence of two given forms which is superior to Minkowski's criterion involving concomitant forms. The results are similar to unpublished results obtained in a different way by Gordon Pall. There are applications to problems involving genera of forms. (Received July 7, 1944.)

204. Gordon Overholtzer: *A new application of the Schur derivate.*

I. Schur introduced the derivate of a sequence $\{a_n\}$ with respect to the number p, $\Delta a_n=(a_{n+1}-a_n)/p^{n+1}$, to generalize Fermat's theorem of elementary number theory. Fermat's theorem yields that Δa^{p^n}, the first Schur derivate of $\{a^{p^n}\}$, is integral; Schur proved the first $p-1$ derivates integral. Let $S[n, g(x)]=\sum'^{p^n}_{i=1} g(i)/p^n$, where the summation is over numbers prime to p, and p is a rational prime. An established number theory result is that $S[n, x^k]$ (k an integer) is integral if $p-1$ does not divide

k and has exponent of p of -1 if $p-1$ divides k. This paper generalizes this result, proving that all the Schur derivates $\Delta^m S[n, x^k]$ are p-adically bounded with exponent of p not less than $-2m-1-m/(p-1)$ and hence p-adically convergent. Formulas for $\lim_{n\to\infty} \Delta^m S[n, x^k]$ are given in terms of $\lim_{n\to\infty} S[n, x^{k-m}]$. For positive k, $\lim_{n\to\infty} S[n, x^{2k+1}]=0$ and $\lim_{n\to\infty} S[n, x^{2k}]=(-1)^{k-1} B_k(1-p^{2k-1})$, where B_i is the ith Bernouillian number. The Schur derivates of $\{S[n, f(x)]\}$, where $f(x)=\sum_{i=2}^{\infty} a_i x^i$ and the valuation of $a_i \to 0$ as $i\to\infty$, are p-adically bounded and convergent; moreover $\lim_{n\to\infty} S[n, f(x)]=\sum_{i=2}^{\infty} a_i \lim_{n\to\infty} S[n, x^i]$. (Received June 29, 1944.)

205. Gordon Pall: *Note on factorization in quadratic fields.*

It is proved that if the quadratic integer $x_0+x_1\omega$ is primitive, that is x_0 and x_1 are coprime, then the divisors of $x_0+x_1\omega$ of a given norm are uniquely determined up to a unit factor. Conditions are obtained for the existence of factors of a given norm. It is claimed that the necessity for the introduction of ideals should be based not on the statement that factorization is not unique, but rather that factors do not exist. Thus in the arithmetic of ordinary quaternions, factorization of imprimitive quaternions is not unique, but that of primitive quaternions is both possible and unique; and ideals are in that case unnecessary. (Received July 10, 1944.)

206. R. R. Stoll: *Primitive semigroups.*

Let F denote the class of semigroups S each of whose elements s satisfies an equation of the form $s^n=s^m$ $(n>m)$. A semigroup $S\in F$ is called primitive if for each idempotent $e\in S$ there exists no idempotent $f\neq e$ such that $ef=fe=f$. Examples of such semigroups are (a) semigroups of F which contain only one idempotent and (b) semigroups containing a zero and such that each element is nilpotent (nil semigroups). The following structure theorem is proved for primitive semigroups. A primitive semigroup S contains a unique minimal ideal M with these properties: it is a completely simple semigroup without zero (Rees, Proc. Cambridge Philos. Soc. vol. 36 (1940) pp. 387–400), and the difference semigroup of S modulo M is a nil semigroup. Conversely, a semigroup $S\in F$ with this structure is primitive. (Received July 10, 1944.)

ANALYSIS

207. R. P. Agnew: *Abel transforms of Tauberian series.*

Let $\rho_1=.9680448\cdots$; the constant is Euler's constant plus log log 2 minus $2Ei(-\log 2)$. The following assertion is true when $\rho\geqq\rho_1$ and false when $\rho<\rho_1$. Let $u_0+u_1+\cdots$ be a series satisfying the Tauberian condition $n|u_n|<K$. Let L be the set of limit points of the sequence of partial sums of $\sum u_n$. Let $\sigma(t)=\sum t^k u_k$ be the Abel transform of $\sum u_n$. Let L_A denote the set of limit points of $\sigma(t)$; $z''\in L_A$ if there is a sequence t_n such that $0<t_n<1$, $t_n\to 1$, and $\sigma(t_n)\to z''$. To each $z'\in L$ corresponds a $z''\in L_A$ such that $|z'-z''|\leqq\rho \limsup n|u_n|$. (Received July 19, 1944.)

208. R. P. Agnew: *A genesis for Cesàro methods.*

The family C_r of Cesàro methods of summability, $r\neq 0, -1, -2, \cdots$, is and can be defined as the unique class of methods of summability whose members are simultaneously Nörlund methods and Hurwitz-Silverman-Hausdorff methods. The only methods simultaneously Riesz methods and Hurwitz-Silverman-Hausdorff methods are methods Γ_r closely related to the methods C_r. (Received June 16, 1944.)