

BOOK REVIEWS

Differentialgeometrien in den Kugelräumen. Vol. II. *Laguerresche Differentialkugelgeometrie.* By T. Takasu. Tokyo, Maruzen, 1939. 20+444 pp.

This volume is the second of a series of three, in which Professor Takasu attempts to give a unified and systematical representation of his work in the differential geometry of sphere space. Concentrating on the geometries of Möbius, Laguerre, and Lie, he presents in this volume his results in Laguerre geometry. In his exposition he follows as closely as possible the usual pattern of euclidean differential geometry, to the systematic study of which the author has contributed in several previous papers.

Following this scheme of development of his subject, Professor Takasu has divided his field into five parts. The first chapter explains the fundamental elements and the coordinates of Laguerre geometry, and shows how we can obtain a correspondence between euclidean three space and a Laguerre plane by means of minimal (isotropic) projection. The group of congruent transformations in space corresponds, in this transformation, to the Laguerre group of the plane. Then follows, in Chapter II, the Laguerre generalization of plane euclidean theory of curves, which is the theory of Laguerre invariants of oriented circles. In Chapter III we find the analogous theory of oriented spheres, corresponding to space curves in ordinary geometry. The Laguerre analogues of surfaces are congruences of spheres, which are discussed in Chapter IV. The final chapter deals with systems of cones, which correspond to line congruences in ordinary space.

This gives us quite a good picture of the structure and of the content of Laguerre differential geometry. The author has placed all previous theories, as those of Darboux, of Ribaucour, of Study, in their proper place and has enriched his work with the investigations of Blaschke and his school. He has shown how much more there is to Laguerre geometry than we could have expected from the study of even the richest other author, and how his methods allow the systematic penetration of all fields into which ordinary differential geometry has thrown its light. We mention, as examples out of a great many other results, the author's treatment of "*L*-developables" ("*L*-Torsen"), of *L*-minimal surfaces, and of Bonnet's plane coordinates. He also ventures into differential geometry in the large, as in a Laguerre version of the four vertex theorem.

We believe that Professor Takasu would have increased the useful-

ness of his work, if he had dwelled longer on the elementary material of his first chapter. The reader who tackles his book without previous knowledge of Laguerre geometry will find the going hard at the beginning. Once he has overcome the initial difficulties, however, he will be richly rewarded by the great number of beautiful results which fall into his lap, and by the mastery of a method which will allow him to find many more results by his own effort.

The book has a complete bibliography and an excellent index.

D. J. STRUIK

The theory and applications of harmonic integrals. By W. V. D. Hodge.

Cambridge, University Press; New York, Macmillan, 1941. 9+281 pp. \$4.50.

This is one of those books which everyone who specializes in a particular branch of group theory, of the theory of algebraic surfaces, of the theory of Riemann surfaces, of topology or of the tensor analysis should consult. It shows how all these different fields are connected, and not connected in some superficial way or in the form of an analogy, but in an essential manner, so that interesting and profound theorems in one field cannot be understood without a thorough knowledge of other fields. In reading this book one is reminded of books like Klein's "Ikosaeder," which is also a blend of several important fields. The task of the reviewer of such a book is hard, because he has seldom the enviable mastery of the different branches of mathematics which the author possesses. At the same time he must praise the author for the beautiful exposition of so many and different fields.

There are chapters on Riemannian manifolds, on integrals and their periods, on harmonic integrals, on their applications to algebraic varieties and on their applications to the theory of continuous groups. The first chapter, on Riemannian manifolds, is divided into a part on tensor calculus and into a part on the topology of such manifolds. All these topics are prepared in a very careful way, with precautions which will satisfy strict demands of rigor. There are references at the end of each chapter.

To indicate the contents of this book by simply copying the chapter headings is to do a great injustice to the author and his work. There is a leading thought in the choice of subject matter, and that is the study of harmonic integrals. Harmonic integrals, however, are not the capricious invention of an imaginative scholar. They appear quite naturally in the generalization of the problems set by