author. Some of the remarks are given above. I do not quite share his opinion that the "methodologist" has only to describe the methods used by the physicists or the scientists in general. Even a non-expert may see sometimes, by general considerations, ways which the researchman should go. Plato and Aristotle were no mathematicians, Bacon no physicist in a proper sense, but, undoubtedly, they furthered the development of mathematics and physics. That Aristotle was an impediment to the development of physics was the fault of "experts" who adhered in a slavish way to his physical theories. The outsider *sometimes* sees more of the general landscape of science, where the scientific workers go the toilsome ways which lead to discoveries.

In his conclusions the author presents certain "indefeasible facts" of the scientific situation, for instance that in modern physics metaphysical explanatory theories are excluded. His last "conclusion" is somewhat metaphysical and not easily accepted, namely that science approaches "asymptotically" a perfectly adequate account of reality. And it is given as an "indefeasible fact of the human situation—that the Human Spirit is one and that the different activities in which it expresses itself must in the end arrive at the same conclusion." It is difficult for me to imagine common conclusions which will be reached "asymptotically" by the Human Spirit in its metaphysical, social and scientific activities.

Only one little critical remark: the mathematical reader will be astonished to find on page 94, quoted from Poincaré, an erroneous description of the method of mathematical induction.

M. Dehn

Transients in linear systems, studied by the Laplace transform. Vol. I. By M. F. Gardner and J. L. Barnes, New York, Wiley; London, Chapman and Hall, 1942. 9+389 pp. $5.00.

This book is remarkable in that it is perhaps the first serious attempt to present the theory of the Laplace transform to the mathematics or engineering student at an early stage of his studies. For the complete understanding of this transform it is necessary to have mastered the theory of functions of a complex variable. This fact has hitherto barred many a student from the use of a valuable analytic tool until rather late in his mathematical career. The present work shows conclusively that this delay is unnecessary.

The authors are able to place the fundamental idea of the method before the reader even in the first chapter. The essence of the matter consists in replacing by the Laplace transform one function space by another in such a way that differential and integral operations on the functions of the first space become algebraic operations on the functions of the second. Then an integrodifferential equation becomes an algebraic equation. Let us illustrate by finding that solution of the differential equation $dy/dx-y=x$ for which $y(0)=0$. One first defines the Laplace transform $Y(s)$ of the required solution $y(x)$:

$$Y(s) = \int_0^\infty e^{-sx}y(x)dx.$$

Integration by parts shows that

$$\int_0^\infty e^{-sx}y'(x)dx = -y(0) + sY(s),$$

so that the given differential equation transforms into

$$sY(s) - y(0) - Y(s) = s^{-2}.$$

After introducing the boundary condition $y(0)=0$ one easily solves the algebraic equation and finds that

$$Y(s) = (s-1)^{-1} - s^{-1} - s^{-2}.$$

It now only remains to find the function $y(x)$ which has this function for its transform. This is easily read off from a table of transforms and is found to be $y(x)=e^x-x-1$.

A critical examination of the above solution will show that certain assumptions have been made, notably that the unknown function $y(x)$ and its derivative have Laplace transforms. The authors take the perfectly acceptable point of view that such assumptions are justified by the results. It is always possible to check directly the solution of a differential system. If it is correct, no further investigation of the method is necessary. If the brilliant success arouses the curiosity of the student, so much the better. He can learn why it all works when he is better equipped with mathematical tools. One outstanding advantage of the method is brought out in the above example. Due to the fact that the transform is unilateral (the integration runs from 0 to $+\infty$ and not from $-\infty$ to $+\infty$), one-point boundary conditions are brought automatically into the solution. Thus no general solution of the differential equation with arbitrary constants is involved.

There is a very welcome second chapter in which the physical background of the electrical and mechanical problems treated is discussed in great detail. Most comparable texts assume that the student is already familiar with the physics of the situation, and if he is not, he must look elsewhere for help. This chapter is independent of the Laplace method and could be used in connection with any course where practice in setting up differential systems from physical problems is required.

It is natural that the authors should wish, in a book of this character, to keep the proof of theorems a secondary matter. Thus in chapter 3 the Laplace transform is introduced by analogy with the more familiar Fourier series and integrals. Even in chapters 4 and 5, where a more formal approach is undertaken, demonstrations of theorems are either omitted entirely (Theorems 1, 2, 3 and 4) or sketched briefly. This procedure seems not to detract from the essential clarity of the presentation. The remainder of the book, with the exception of chapter 8, elaborates further and illustrates profusely the method of solving integrodifferential and difference equations.

The reviewer's chief adverse criticism of the book has to do with chapter 8. Here the authors seem to abandon the general plan of the book. Some twenty theorems of a more advanced nature about the Laplace transform are developed. Demonstrations are given. No use of the results is made in the present volume. In several cases the theorems are incorrectly stated, in others the proofs are wrong or at least inadequate. One of the more important errors stems from indiscriminate interchange of limit and integral operations (though elsewhere the authors are careful about this). Another is based on the incorrect assumption that a Laplace integral defines an analytic function which must have a singularity on the axis of convergence. No doubt the chapter was included to prepare the way for a second volume on partial differential equations and related material which is to follow. The flaws in the chapter need not impair essentially the usefulness of the present volume. For, the student who is looking merely for technique may omit the chapter, the brilliant student may read it with circumspection. It may be worth while to call attention to one other misstatement in chapter 4. Here it is stated that a function of a complex variable which has a first derivative at a point necessarily has all higher derivatives there also (compare the function $|z|^2$).

In conclusion it may be well to summarize the special features of the book. First and foremost, it places a vital tool well within the grasp of a student with meager mathematical training. It dispells the mystery of the Heaviside operational calculus by showing the real basis of that technique. It gives complete physical background for the integrodifferential and difference equations it solves. It has a wealth of illustrative examples, done in the text, and many problems for the student at the ends of the chapters (no answers). There is a very extensive table of Laplace transforms, as useful as a table of integrals in a calculus course. In chapter 1 and in appendix B a valuable comparison of the Laplace method with other possible techniques is given. Historical notes on the mathematical theory appear in appendix C. Finally there is one of the most extensive bibliographies on the subject yet to appear.

D. V. WIDDER

A treatise on projective differential geometry. By Ernest Preston Lane. Chicago, University of Chicago Press, 1942. 9+466 pp. $6.00.

Since the appearance of the author's earlier volume, *Projective differential geometry of curves and surfaces* (University of Chicago Press, Chicago, 1932), significant contributions to the field of projective differential geometry have been made by geometers in various parts of the world. In the preface the author does not claim that the present treatise is exhaustive, but states that it represents the fruit of ten years of study and investigation and gives an account of the author's experience with those portions of the subject which interested him most. It is with respect to those portions of the subject, then, that the reviewer interprets the author's statement, appearing earlier in the paragraph, that "the present volume integrates the new material with the old and gives a connected exposition of the theory to date." The treatise reports results of studies made by many workers in the field, and expounds a wide range of topics. It is also notable, however, that some of the newer topics that have attracted rather general interest have not been mentioned. Parts of the proofs, involving calculations that are difficult for the uninitiated, are so frequently left to the care of the reader that only the more advanced students of the subject can use the volume successfully as a textbook. The treatise is properly designed to serve as a reference book for the research worker in the field. Geometric concepts and results are consistently described in a lucid graphic manner.

It is satisfying to observe that the present volume devotes considerably more attention to the methods of Wilczynski on the study of curves by means of linear differential equations than does the earlier volume. It is disappointing, however, to find no mention made of Stouffer's simplifications of Wilczynski's methods of determining (i) canonical power series expansions for the local equations of plane and space curves, and (ii) the geometric characteriza-