It is known that the asymptotic osculating quadrics at a point of a curve on a surface coincide if and only if the curve is tangent to a curve of Darboux and the surface satisfies the relation (1) $\beta\psi^3=\gamma\phi^3$. This paper investigates the properties of surfaces which satisfy this relation indentically. In particular coincidence surfaces possess all of these properties. There exist, however, surfaces satisfying the relation (1) which are not coincidence surfaces. A necessary and sufficient condition that a surface which satisfies (1) be also a cubic surface is given and used to prove that there is, in the sense of projective equivalence, only one cubic coincidence surface. (Received March 11, 1943.)

Logic and Foundations

167. B. A. Bernstein: *Postulate sets for Boolean rings.*

The author gives nine sets of postulates for Boolean rings in terms of ring operations. Each set is independent, and remains independent when a unit-element postulate is added. (Received March 25, 1943.)

168. Max Zorn: *Informal note on the second underivability theorem.*

The content of this note may be condensed into one question: In which sense does a formula like (Ex) (Form (x) & $\overline{\text{Bew}}$ (x)), which from the formalist point of view has no independent meaning, "represent" the consistency of a formalism in the sense of Hilbert? The answer to this question is expected from those who insist that the underivability of such formulae constitutes evidence in support of the opinion that finitary consistency proofs of the type which so far have been employed by the Hilbert school probably cannot be found for the arithmetic formalism or *Principia Mathematica*. (Received March 27, 1943.)

Statistics and Probability

169. Leon Alaoglu: *Harmonic analysis of stochastic processes.* Preliminary report.

If Ω is a differential stochastic process of elements $z(t)$ which are complex-valued functions of a real variable t, such that the distribution function of the variable $e^{i\theta}(z(t+h)-z(t))$ is independent of θ (θ real) and if the expectations $F(t)=\int_\Omega|z(t)-z(0)|^2dP$, $m(t)=\int_\Omega(z(t)-z(0))dP$ exist, the first being bounded and the second vanishing, then the Fourier transform of the function $z(t)$ exists for almost all z and defines a stationary stochastic process. (Received March 26, 1943.)

Topology

170. B. H. Arnold: *On decompositions of T_1 spaces.*

Several authors (Banach, *Théorie des Opérations Linéaires*, p. 170; Eilenberg, Ann. of Math. vol. 43 (1942) pp. 568–579; Eidelheit, Studia Mathematica vol. 9 (1940) pp. 97–105) have proved theorems of the form: The "structure" of a certain class of transformations defined on a suitable space A to a fixed suitable space B determines the space A. In the present paper the author proves an analogous result which is valid for a very wide class of spaces A, but at the expense of allowing B to become variable. If two T_1 spaces, A, A' are such that the ordered system M of the upper semicontinuous decompositions of A is isomorphic to that of A', then A and A' are homeomorphic. Separation, connectedness, and compactness properties of the space

A are expressed in terms of order properties of M and a procedure is given which allows A to be constructed from M, again using only order properties of M. (Received March 6, 1943.)

171. D. G. Bourgin: *On a quasi linear operation.*

Let E be a linear topological space which is convex (though this last restriction can be weakened). The operators referred to below are on a subset of E to E, satisfy $U(\alpha_1 x + \alpha_2 y) = \alpha_1 U(x) + \alpha_2 U(y)$ for $\alpha_i \geqq 0$, $\alpha_1 + \alpha_2 = 1$, and are continuous. A typical result is that if ϕ induces an automorphism of the convex bicompact set $K \subset E$ then the relation $\phi = (\psi + \gamma)/2$ where $\psi(K) = K$ implies $\gamma = \psi = \phi$. An incidental result is that under certain fairly general conditions U may be extended to U' on the closed linear extension of the original domain and then satisfies $U'(ax+by) = aU'(x) + bU'(y) + (a+b-1)U'(\theta)$ and is merely a translation plus a linear transformation. (Received March 26, 1943.)

172. R. H. Fox: *Natural systems of homomorphisms.* Preliminary report.

Let P_{3k+1} denote the kth homotopy group of a complex Y, P_{3k+2} the kth homotopy group of the n-dimensional skeleton X, and P_{3k} the kth homotopy group of Y mod X. Let Q_{3k+1}, Q_{3k+2} and Q_{3k} denote the corresponding continuous homology groups with integer coefficients. The groups P_m, Q_m, together with the natural homomorphisms $r = r_m^{(P)}$ of P_m into P_{m-1}, $r = r_m^{(Q)}$ of Q_m into Q_{m-1}, and $h = h_m$ of P_m into Q_m constitute the natural system of Y, X. The nucleus of r_m is the image of r_{m+1}, and $hr = rh$. These two simple facts lead to the following theorem: If Y is simply connected and acyclic in the dimensions less than or equal to $n-2$, then every n-cycle is spherical. Other consequences are a generalization of a theorem of S. Eilenberg (Bull. Amer. Math. Soc. vol. 47 (1941) p. 432) and the recent theorem of H. Hopf (Comment. Math. Helv. vol. 14 (1942) p. 257). (Received March 25, 1943.)

173. R. C. James: *Orthogonality in normed linear topological spaces.*

The purpose of this paper is to investigate certain possible definitions of orthogonality in normed linear topological spaces, properties possessed by elements orthogonal in each sense, and conditions that each type of orthogonality have certain common and desirable properties. These properties are: (1) If x is orthogonal to $y (x \perp y)$, then $y \perp x$; (2) if $x \perp y$, then $ax \perp by$ for all a and b; (3) if $x \perp y$ and $x \perp z$, then $x \perp (ay+bz)$ for all a and b; (4) for any elements x and y there exists a number a such that $x \perp (ax+y)$. The three definitions of orthogonality which are used are: (1) $x \perp y$ if $\|x-y\| = \|x+y\|$; (2) $x \perp y$ if $\|x\|^2 + \|y\|^2 = \|x-y\|^2$; (3) $x \perp y$ if $\|x+ay\| \geqq \|x\|$ for all a. All three types of orthogonality have the fourth property, while it is shown that if orthogonality of either of the first two definitions has all four properties the norm can be defined by a bilinear and symmetric inner product. All three definitions are equivalent and have all four properties when the norm is given by such an inner product. (Received March 12, 1943.)

174. Tibor Radó: *On continuous path-surfaces of zero area.*

Let S: $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$, $(u, v) \in Q$: $0 \leqq u \leqq 1$, $0 \leqq v \leqq 1$, be a continuous path-surface. Let $[S]$ denote the point-set in (x, y, z) space that corresponds to Q by means of the equations of S. The Lebesgue area $A(S)$ of S is not determined by

$[S]$, but rather by the manner in which $[S]$ is described while (u, v) describes Q. In fact, simple examples, due to Geöcze, show that $A(S)$ may equal zero even though $[S]$ is a solid cube. Geöcze derived various necessary and sufficient conditions for $A(S)=0$. The purpose of this paper is to establish further conditions of this type. The methods used are essentially topological. (Received February 19, 1943.)

175. G. E. Schweigert: *Fixed elements and periodic types for homeomorphisms on semi-locally-connected continua.*

Let S be a semi-locally-connected continuum and $T(S)=S$ a homeomorphism. If $N\neq S$ is an invariant node then there exists in S another invariant cyclic element $E\neq N$. This leads to equivalence of five properties of Ayres and Whyburn (cf. Whyburn, *Analytic topology*, Amer. Math. Soc. Colloquium Publications vol. 28, 1942, chap. 12, Theorems 4.21–4.5); pointwise almost periodicity is disregarded, second property "localized" to invariant A-set. Componentwise periodicity means, for every true invariant A-set A, each component of $S-A$ has finite period. In order that $T(S)=S$ be elementwise periodic on cyclic elements of complement of end points of S it is necessary and sufficient that one of the following conditions holds: (a) T is componentwise periodic, and for every positive integer n, T^n has one of five properties above; (b) T is componentwise periodic and each T^n-invariant A-set A, with no points of A on true cyclic element of $S-A$, admits a contracting T^n-approximation at A, that is, for every A-set C with $A\subset \text{Int } C$ there is a third T^n-invariant A-set B such that $A\subset \text{Int } B\subset C$; (c) Theorem (4.7), chap. 12, holds. Part (b) is applied to give a certain continuous orbit decomposition. (Received February 27, 1943.)

176. G. E. Schweigert: *Minimal A-sets, infinite orbits, and fixed elements.*

If E is a cyclic element of a semi-locally-connected continuum S with infinite period relative to the homeomorphism $T(S)=S$ and B is the least invariant A-set which contains the orbit of E, then $B=C(A, D)+0$, where 0 is the closure of the orbit of a cyclic chain $C(E, Y)$ with $C(E, Y)\cdot C(A, D)=Y$ and A and D are the invariant cyclic elements of B determining the cyclic chain $C(A, D)$. If $A\neq D$, then A and D are fixed end points of $C(A,D)$; if $A=D$, then $C(A,D)=Y$ is an unique invariant cyclic element called center of rotation. If $E=Y$, then $B=C(A,D)$. As a corollary it is sufficient to get the Ayres' property (Whyburn, *Analytic topology*, Amer. Math. Soc. Colloquium Publications vol. 28, 1942, chap. 12, Theorem 4.5) to assume $C(E, T(E))$ contains an invariant element in (and only in) case E has an infinite orbit. If $T(S)=S$ is elementwise periodic on all cyclic elements except the end points of S, the least invariant A-set M containing the infinite orbit of an end point p of S has an unique invariant element, and the dendritic form of the cyclic element hyperspace of M is similar to the dyadic tree. (Received February 27, 1943.)

177. J. W. T. Youngs: *On parametric representations of surfaces.* III.

A representation is a continuous transformation $R(A)=B$ from the 2-sphere A to a set B in 3-space. $R(A)=L(M(A))$, where $M(A)=\Sigma$ is monotone and $L(\Sigma)=B$ is light. Σ is imbedded in 3-space so that R is of degree 1 on each true cyclic element of Σ. Two representations R_1 and R_2 are F^+ equivalent if the Fréchet distance between them is zero and the homeomorphisms of the definition are of degree $+1$. They are K^+ equivalent if there is a homeomorphism $H(\Sigma_1)=\Sigma_2$ such that $L_1(\Sigma_1)=L_2(H(\Sigma_1))$

and H is of degree $+1$ on each true cyclic element of Σ_2. F^- and K^- equivalence are defined similarly. R_1 is Fréchet equivalent to R_2 if R_1 is either F^+ or F^- equivalent to R_2, and similarly for K equivalence. Various relations between these definitions are investigated. Applications to the theory of area are made on the basis of the fact that F equivalence is the same as K equivalence. A general method for obtaining smooth representations is developed. The content of the paper is examined from the point of view of the μ-length of Morse. (Received March 26, 1943.)