logical product of a $(3N-1)$-sphere and a $3N$-sphere, minus certain $(6N-4)$-dimensional loci corresponding to collisions. For $C \leqq -\delta$ the structure of $M(C)$ has yet to be determined. It is planned to correlate the critical values $C=0$ and $C=-\delta$ with the physically known transition temperatures such as the critical temperature. (Received March 23, 1943.)

159. Isaac Opatowski: *An explicit formula for the refractive index in electron optics.*

The refractive index μ is expressed in electron optics (W. Glaser, Zeitschrift für Physik vol. 81 (1933) pp. 647–686) in terms of the electrostatic potential V, the magnetic vector potential $\boldsymbol{A}$ and the unit vector $\mathbf{s}$, which is defined as tangent to the electron trajectory. Since $\mathbf{s}$ is not known a priori and is a function of V and $\boldsymbol{A}$, the elimination of $\mathbf{s}$ from the expression of μ is of advantage. This is done in the paper for a very ample class of fields in which a momentum integral of the equations of motion exists (Bull. Amer. Math. Soc. vol. 46 (1940) p. 887 and Journal of Mathematics and Physics vol. 20 (1941) pp. 418–424). (Received March 26, 1943.)

Geometry

160. Jesse Douglas: *Point transformations and isothermal families of curves.* II.

This paper is a continuation of one with the same title (see Bull. Amer. Math. Soc. abstract 49-1-71). Its new feature is the principal use of synthetic rather than analytic methods. The problem is referred to the investigation of certain properties of a hexagonal web. (Received February 27, 1943.)

161. Jacques Dutka: *Transversality in higher space.*

In this paper, a geometric criterion for transversality developed by Kasner in his paper *Transversality in space of three dimensions* (Trans. Amer. Math. Soc. vol. 30 (1928) pp. 447–452) is generalized for n-dimensional Euclidean space. It is shown here that a necessary and sufficient condition for a given correspondence between a lineal element and a hypersurface element to be a transversality is that a certain induced correlation be a polarity. A principle of transference connecting simple and $(n-1)$-fold integrals in the calculus of variations when they produce equivalent transversalities is established. The result obtained is applied to the theory of infinitesimal contact transformations from which are derived analytic tests equivalent to the above-mentioned geometric criterion. Actual examples of transversalities in addition to the well known condition of orthogonality are also given. (Received March 25, 1943.)

162. Jacques Hadamard: *On fractional iteration and connected questions.*

The author presents some results communicated to him by two younger geometers on fractional iteration and permutable transformations in one variable. This subject is connected with group theory or, more precisely, with Cartan's conception of geodesics in a group-space. (Received March 27, 1943.)

163. T. R. Hollcroft: *Plane curve systems with distinct nodes and cusps and of negative virtual dimension.*

In this paper a system C of plane curves is defined by plane sections of a tangent cone of species $r-2$ from an S_{r-3} to a nonsingular, irreducible primal V_{r-1} of order ν in S_r. The only singularities of C are distinct nodes and cusps. The system C has a negative virtual dimension for certain limiting values of r and ν. The above has been presented to the Society (Bull. Amer. Math. Soc. abstract 44-11-412). At that time, the system C had not been proved irreducible. The irreducibility of C has been established. These systems are at present the only plane curve systems known to exist of negative virtual dimension with no other singularities than distinct nodes and cusps. (Received March 26, 1943.)

164. Edward Kasner and John DeCicco: *Generalized transformation theory of isothermal families.*

Kasner has proved that the complete group of lineal-element transformations which send every isothermal family of curves into an isothermal family is the product of the conformal group by the non-contact group $U=u$, $V=v$, $\Theta=a\theta+h(u)+k(v)$. (Note that (u, v) are the minimal coordinates of the point and θ is the inclination.) Further generalizations have been given by the authors to first order field-element transformations and curvature-element transformations (Proc. Nat. Acad. Sci. U.S.A. vol. 27 (1941) pp. 406–412, vol. 28 (1942) pp. 52–55 and pp. 328–338). In this paper, the authors determine all nth order field-element to lineal-element transformations which preserve the isothermal character. Denote $r_\alpha=\partial^\alpha\theta/\partial u^\alpha$, $t_\beta=\partial^\beta\theta/\partial v^\beta$, $s_{\alpha\beta}=\partial^{\alpha+\beta}\theta/\partial u^\alpha\partial v^\beta$ for $\alpha, \beta=1, 2, \cdots, n$. Any transformation of our set must be of the form $U=\phi(u, r_\alpha, s_{\alpha\beta})$, $V=\psi(v, s_{\alpha\beta}, t_\beta)$, $\Theta=a\theta(s_{\alpha\beta})+h(u, r_\alpha, s_{\alpha\beta})+k(v, s_{\alpha\beta}, t_\beta)$, or this type followed by a reflection in the X-axis. Finally it is shown that the total set of all nth order differential-element to lineal-element transformations preserving the class of all isothermal families is exactly the Kasner group. (Received March 23, 1943.)

165. Edward Kasner and John DeCicco: *The congruence of element-series associated with a polygenic function.*

The derivative $\gamma=dw/dz$ of the polygenic function w with respect to z induces a correspondence T between the lineal-elements (x, y, θ) of the z-plane and the points $\gamma=\alpha+i\beta$ of the γ-plane. This associated transformation T carries the ∞^1 elements at a point in the z-plane into a circle (the Kasner circle) in the γ-plane. However, a point in the γ-plane does not correspond to a single element but to a series of elements. Therefore, in general, there exists a congruence (∞^2) of series in the z-plane which by T is converted into the set of ∞^2 points in the γ-plane. Kasner has already determined those polygenic functions for which the associated ∞^2 series are all unions. In that event, the unions are all circles passing through a fixed point (Bull. Amer. Math. Soc. vol. 44 (1938) pp. 726–732). In this paper, a complete analytic characterization of the congruence of element-series associated with any polygenic function is given. One geometric property is that, if the tangent turbines of the ∞^1 series which pass through a given point z are constructed at z, the centers of these turbines will describe a conic section. (March 23, 1943.)

166. J. E. Wilkins: *A special class of surfaces in projective differential geometry.*

It is known that the asymptotic osculating quadrics at a point of a curve on a surface coincide if and only if the curve is tangent to a curve of Darboux and the surface satisfies the relation (1) $\beta\psi^3=\gamma\phi^3$. This paper investigates the properties of surfaces which satisfy this relation indentically. In particular coincidence surfaces possess all of these properties. There exist, however, surfaces satisfying the relation (1) which are not coincidence surfaces. A necessary and sufficient condition that a surface which satisfies (1) be also a cubic surface is given and used to prove that there is, in the sense of projective equivalence, only one cubic coincidence surface. (Received March 11, 1943.)

Logic and Foundations

167. B. A. Bernstein: *Postulate sets for Boolean rings.*

The author gives nine sets of postulates for Boolean rings in terms of ring operations. Each set is independent, and remains independent when a unit-element postulate is added. (Received March 25, 1943.)

168. Max Zorn: *Informal note on the second underivability theorem.*

The content of this note may be condensed into one question: In which sense does a formula like (Ex) (Form (x) & $\overline{\text{Bew}}$ (x)), which from the formalist point of view has no independent meaning, "represent" the consistency of a formalism in the sense of Hilbert? The answer to this question is expected from those who insist that the underivability of such formulae constitutes evidence in support of the opinion that finitary consistency proofs of the type which so far have been employed by the Hilbert school probably cannot be found for the arithmetic formalism or *Principia Mathematica*. (Received March 27, 1943.)

Statistics and Probability

169. Leon Alaoglu: *Harmonic analysis of stochastic processes.* Preliminary report.

If Ω is a differential stochastic process of elements $z(t)$ which are complex-valued functions of a real variable t, such that the distribution function of the variable $e^{i\theta}(z(t+h)-z(t))$ is independent of θ (θ real) and if the expectations $F(t)=\int_\Omega|z(t)-z(0)|^2dP$, $m(t)=\int_\Omega(z(t)-z(0))dP$ exist, the first being bounded and the second vanishing, then the Fourier transform of the function $z(t)$ exists for almost all z and defines a stationary stochastic process. (Received March 26, 1943.)

Topology

170. B. H. Arnold: *On decompositions of T_1 spaces.*

Several authors (Banach, *Théorie des Opérations Linéaires*, p. 170; Eilenberg, Ann. of Math. vol. 43 (1942) pp. 568–579; Eidelheit, Studia Mathematica vol. 9 (1940) pp. 97–105) have proved theorems of the form: The "structure" of a certain class of transformations defined on a suitable space A to a fixed suitable space B determines the space A. In the present paper the author proves an analogous result which is valid for a very wide class of spaces A, but at the expense of allowing B to become variable. If two T_1 spaces, A, A' are such that the ordered system M of the upper semicontinuous decompositions of A is isomorphic to that of A', then A and A' are homeomorphic. Separation, connectedness, and compactness properties of the space