of presentation, since in the long run it is more economical as far as "economy of thought" is concerned.

Ths book is divided into three chapters: Chapter I, Spaces, Chapter II, Transformations, and Chapter III, Orthogonality. There are also three appendices: on the classical canonical form, on the direct products, and on Hilbert space. This last appendix is just a short preview of what the reader will get into when he takes up further study.

The chapters are divided into sections, and this division was done with great care and didactical skill. The headings of the sections indicate by themselves the impressive variety of topics treated. These range from the relatively familiar ones like linear dependence (§§4, 5), linear manifolds (§9), or proper values of Hermitean and unitary transformations (§62), to those that are of a definitely less familiar and more difficult character like reflexivity of finite dimensional spaces (§15), polar decomposition (§67), or the ergodic theorem for unitary transformations (§76).

However, the reviewer doubts whether this book could be used successfully in a course on matrices and linear equations. The presentation is definitely that of an analyst and the "algebraic" point of view is purposely avoided. The author states in the preface that his purpose is " . . . to emphasize the simple geometric notions common to many parts of mathematics, and to do it in a language which gives away the trade secrets and tells the students what is in the back of the minds of people proving theorems about integral equations and Banach spaces."

The reviewer thinks that the author succeeded admirably in this respect.

MARK KAC

The non-singular cubic surfaces. By B. Segre. Oxford, The Clarendon Press, 1942. 11+180 pp. 63 figures. $4.50.

The extensive literature on the cubic surface would hardly justify further additions unless such additions contributed a novel approach. The novelty of the approach in the case of the volume under review justifies its inclusion.

A non-singular cubic surface F and a triad of planes not belonging to a pencil form a pencil the surfaces of which remain non-singular in all the intermediate positions from F to the triad of planes. Under these circumstances the lines on the surface are always distinct and preserve their incidence relations, thus giving rise to the group.

The book is divided into four chapters devoted to (1) the discussion of the lines and the configurations formed by them, (2) the group, (3) the properties of the surface under the restriction that it shall be real, with all possibilities of real and imaginary lines, the latter occurring in pairs, (4) the exceptional cases of the pentahedroid.

In the limiting triad of planes, intersecting by pairs in three lines, each of which meets F in three points, the 27 lines are represented by joining these points in pairs. Consider a symbol abc, one of which one is zero, and each of the others may be 1 or 2, or 3, independently of each other. The numerical value and the position of each letter in the symbol has meaning. Consider the points 1, 2, 3 on each line r_1, r_2, r_3 in which two planes of the triad intersect. The position of a refers to points on r_1, and so on. Thus, the symbol 012 is the (image) line having no point on r_1, having point 1 on line r_2 and the point 2 on r_3. Two lines abc and $a'b'c'$ intersect when and only when these symbols have one and only one element as $a=a'$, or $b=b'$, or $c=c'$ alike. The configuration of the lines of F can now be expressed in terms of simple combinatorial relations in these symbols and expressed graphically in axonometric projection. Thus, the following theorems are samples. They are immediate consequences of the symbolism.

Every line meets ten others.

Given a line r, and any two others, skew to it and to each other; then three other lines can be found having similar relations among themselves, but each new line meeting all the lines of the first group.

Given any line r, there are five others skew to it and to each other.

The properties of the double sixes and of the tritangent planes follow at once. A plane through a line r meets F in a conic which meets r in two points of an involution, the double points of which on the three lines of any tangent plane lie by threes on the sides of a complete quadrilateral.

The second chapter discusses the group of the 27 lines from the property that incident relations are preserved. It is shown to have six independent generators, directly suggested from the graphical representation. It contains a conjugate set of 36 involutorial operations, called σ-transformations, each of which interchanges the conjugate lines of two complementary sextuples and leaves unaltered the remaining 15. The 15 pairs of parabolic points of the 15 lines of F residual to a double six are conjugate with respect to a Schur quadric of the double six. The six pairs of parabolic points of the lines of any two complementary triplets lie on a space quartic curve of the first kind.

The third chapter—on real non-singular cubic surfaces—is the main theme discussed in the book; it contains about two-thirds of the entire volume. Consider the pencil $F+\lambda\phi=0$, wherein ϕ is a nodal cubic. The six lines of F through the node replace the 12 lines of a double six; the incidence relations of the remaining 15 are those of F. The classification of surfaces F into four distinct types, according to the number of pairs of conjugate imaginary lines, and the addition of one more type with 12 pairs of conjugate imaginary lines, now proceeds as in the Schläfli enumeration, supplemented by topological consideration of the space [19] of all cubic surfaces and the primal representing the discriminant of the surface. It is shown that these five types comprise all possible forms of non-singular real cubic surfaces. Each of these types is then considered in more detail, the graphical representation being applied to each, with special devices for representing the imaginary lines of the first and second kind. Expressed in this way, the configurations become obvious; the various kinds of double sixes are immediately apparent. The groups of the various types are considered further, featuring the differences between right-hand and left-hand lines as determined by the sequence of parabolic planes and the planes of composite residual section, as the plane turns about the line. This study is then applied to the configurations of the real sextuples of lines of each type, and finally to the topology of real cubic surfaces in connection with the real lines. With this preparation the form of the parabolic curve on each type is determined.

The last chapter is analytic. It is devoted to the Sylvester pentahedron and to those cubic surfaces invariant under homographic transformations. The general cubic surfaces can be expressed by an equation involving five cubes of linear forms in one and only one way. The only non-singular cubic surfaces for which there is a point V such that all their sections by planes through V have the same modulus are the cyclic surfaces having V as center. Conversely, all the plane sections of the non-conical cyclic cubic surfaces are equianharmonic. A real cubic surface with 27 distinct real lines has always a well determined Sylvester pentahedron, consisting of 5 distinct real planes, no four of which are dependent.

Three short appendices appear at the end of the book, devoted respectively to the degeneration of a cubic surface into a plane and a quadric, the degeneration of a non-singular cubic primal of [4] into three primes, and on singular surfaces of a pencil of cubic surfaces.

The press-work is of the expected excellence of the Clarendon Press. The 63 figures are particularly good. The volume is strikingly free of typographical errors.

VIRGIL SNYDER