div $u_1=0$ and div $u_2=0$ are retained. u_0 must have wave character with an exceedingly small wave length λ. Such cases can be handled by means of linear differential equations. (Received November 21, 1942.)

68. Brockway McMillan: *Networks of mechanisms.* Preliminary report.

A mechanism M maps a class I of input histories $i(t)$ upon a class O of output histories $o(t)$, $-\infty<t<\infty$. It is single-valued and has the property that whenever $i_1(t)\equiv i_2(t)$ for $t\leqq t_o$, then $o_1(t)\equiv o_2(t)$ for $t\leqq t_o$. If $o_1(t)\equiv o_2(t)$ for $t\leqq t_o+\lambda$, uniformly in i_1, i_2, and t_o, then M has the latency λ. Suppose that I is closed under an operation of addition, that a null function $i_o(t)\equiv\phi$ is in I, that every $i(t)$ is identically ϕ near $t=-\infty$, and that $O\subseteq I$. Let (M_k) be a collection of mechanisms from I to O such that (a) each has latency at least $\lambda>0$, and (b) each maps the null function on itself. The inputs and outputs of the M_k are interconnected to form a network N. Arbitrary inputs $i(t)\in I$ at each junction make the components of a vector input to N. The vector output of N has for its components the outputs of the various M_k. Theorem: I and O can be extended so that N is a mechanism between its vector inputs and outputs, with latency λ. A motivation is the possibility of application to nerve fiber networks. (Received November 4, 1942.)

69. W. H. Roever: *A new formula for the deviation in range of a projectile due to the earth's rotation.*

On page 68 in his monograph entitled, *The weight field of force of the earth*, published in the Washington University Studies, September, 1940, the author derives for the range of a projectile, a formula [second part of (129)] which by a simple trigonometric transformation can be put in the new form $\bar{x}=(v_0^2/g_1)\sin 2\beta+\Delta\bar{x}$ where $\Delta\bar{x}=-(4v_0^3/3g_1^2)\,\omega\cos\phi_1\sin 3\beta\sin\alpha$, in which ω is the angular velocity of the earth's rotation, g_1 is the acceleration, due to weight, at the position of the gun, ϕ_2 is the astronomical latitude of the position of the gun, α is the azimuth (measured from the south through the west) of the direction of fire, β is the angle of elevation of the gun, v_0 is the muzzle velocity of the projectile, he points out particularly that for fixed values of α and ϕ_1, $\Delta\bar{x}$ changes sign when $\beta=60°$. (Received November 23, 1942.)

Geometry

70. John DeCicco: *Conformal geometry of second order differential equations.*

Kasner in his fundamental paper, *Conformal geometry*, Prcoeedings of the International Congress of Mathematicians, 1912, initiated the conformal study of sets of analytic curves. In previous work, Kasner (with the author) studied the conformal geometry of velocity systems of curves $y''=(1+y'^2)[\phi(x, y)+y'\psi(x, y)]$. This class of velocity systems characterizes the conformal group. Any velocity system possesses six absolute conformal differential covariants of second order. In this paper, it is shown that a system of ∞^2 curves, not of the velocity type, possesses three absolute conformal differential covariants of third order. Moreover any other conformal covariant is a function of these and their partial derivatives. Geometric interpretations of these covariants are also obtained. (Received November 21, 1942.)

71. Jesse Douglas: *Point transformations and isothermal families of curves.*

A classification is made of all point transformations $U=f(u, v)$, $V=g(u, v)$ $(u=x+iy$, $v=x-iy)$ with respect to the isothermal families of curves $(\phi(u)+\psi(v)=\text{const.})$ which they convert again into isothermal families. An incidental result is a very simple proof of the theorem that only conformal transformations, $U=f(u)$, $V=g(v)$, convert all isothermal families into isothermal families. (A proof of a more general theorem, involving arbitrary lineal element transformations, has been given recently by Kasner.) Canonical transformations found are (1) $U=u$, $V=u+v$, which preserves the isothermal character of $V+\phi(U)=\text{const.}$ (ϕ arbitrary); (2) $U=u+v$, $V=u-v$, preserving the isothermality of all curve families equivalent by translation and similitude to that defined by $\text{sn } U\,dU+\text{sn } V\,dV=0$. Here sn is Jacobi's elliptic function with any modulus k. Interesting special cases (trigonometric, algebraic) are obtained for particular values of k. The given canonical transformations may be pre- and post-multiplied by arbitrary conformal transformations. The problem may be rendered topological by referring it to the dctermination of the common diagonal curves of two given nets. (Received November 23, 1942.)

72. Edward Kasner: *The close packing of spheres.*

If space is packed with equal spheres the volume occupied is about seventy-five per cent and the porosity about twenty-five per cent. By using many spheres of unequal radii, it is proved that the porosity can be made as near zero as desired. An analogous theorem holds for circles in a plane, or for hyperspheres in any space. (Received October 9, 1942.)

73. Edward Kasner and John DeCicco: *Bi-isothermal systems in pseudo-conformal geometry.*

In this paper, generalizations are obtained of Kasner's theorems on biharmonic functions in the paper, *Biharmonic functions and certain generalizations*, Amer. J. Math. vol. 58 (1936) pp. 377–390. A bi-isothermal system of hypersurfaces in euclidean four-dimensional space is pseudo-conformally equivalent to ∞^1 parallel hyperplanes. Similarly a bi-isothermal system of curves is pseudo-conformally equivalent to ∞^3 parallel lines. Any bi-isothermal system of hypersurfaces is intersected by a conformal surface in an isothermal system of curves. If every bi-isothermal system of hypersurfaces intersects a given surface s is an isothermal system, then s is conformal. If a system S of ∞^1 hypersurfaces is intersected by every conformal surface in an isothermal system, then S is bi-isothermal. Kasner's pseudo-angle between any two bi-isothermal systems of hypersurfaces and curves is a biharmonic function. The complete group of point transformations preserving the class of biharmonic systems of hypersurfaces (or curves) is the mixed pseudo-conformal group. (Received November 21, 1942.)

74. Mary E. Ladue: *Conformal geometry of horn angles of higher order.*

The conformal measures of horn angles (curvilinear angles of zero magnitude) between curves having third, fourth and fifth order contact are obtained. The conformal geometry of the third order horn set (the set of all curves having third order contact with each other at a given point) is studied by making correspond to each curve of the horn set a point in a four-dimensional space K_4 in which the metric is given by the conformal measure of a horn angle of the horn set, and the transformation is that induced on the points of the K_4-space by a conformal transformation on the curves of the horn set. The invariants of the geometry of this K_4-space are then obtained. Finally the equilong measure (see Kasner, *Equilong invariants and convergence proofs*, Bull. Amer. Math. Soc. vol. 23 (1917) pp. 341–347) of horn angles of third order contact is obtained and the resulting equilong geometry of the horn set is shown to be isomorphic to the conformal geometry of the corresponding horn set. Similar results for the equilong geometries of horn sets of fourth and fifth order are to be presented soon. (Received November 20, 1942.)

75. Karl Menger: *Statistical generalizations of metric geometry.*

A probability function $\Pi(x; p, q)$ instead of a number is associated with each ordered pair of elements p, q of a set S and interpreted as the probability that the point p has a distance at most x from the point q. Assume as postulates $\Pi(0; p, q)=1$ or $\neq 1$; according to whether $p=q$ or $p\neq q$; $\Pi(x; p, q)=\Pi(x; q, p)$; and a triangular inequality $\Gamma(\alpha, \beta)\leqq\gamma$ connecting $\alpha=\Pi(x; p, q)$, $\beta=\Pi(y; q, r)$, and $\gamma=\Pi(x+y; p, r)$ where $0\leqq\Gamma(\alpha, \beta)=\Gamma(\beta, \alpha)\leqq 1$ and Γ is a non-decreasing function such that $\Gamma(1, 1)=1$ and $\Gamma(\alpha, 1)>0$ if $\alpha>0$. Define q to be between p and r and write pqr if $\Gamma(1-\alpha, 1-\beta)\leqq 1-\gamma$. Then it can be shown that (1) pqr implies rqp; (2) if q is distant from r, that is, if for some $y>0$, $\Pi(y; q, r)=0$, then pqr and prq are incompatible; (3) under certain conditions, pqr and prs imply pqs and qrs. If S contains more than four elements it can be ordered by means of the above defined betweenness relation. (Received October 28, 1942.)

76. P. M. Pepper: *A new method for imbedding theorems.*

Let Σ be a system of sets with a congruence relation for pairs of elements of Σ, and let E be an element of Σ such that each $n+4$ or more point element of Σ which is not congruent to a subset of E has at least $m+1$ of its $n+3$ point subsets not congruent to subsets of E. Then for $k\geqq 1$, each $n+3+k$ or more point element of Σ which is not congruent to a subset of E has (in terms of the binomial coefficients $C_{n,r}$) at least $1+mk+(m-1)C_{k,2}+\cdots+C_{k,m}$ of its $n+3$ point subsets not congruent to subsets of E. If Σ is the class of all semimetric spaces, the above theorem, in conjunction with a finite covering theorem, permits a new proof of a sharp imbedding theorem for the euclidean n-space under weakened hypotheses and a comparable new result for a convex n-sphere. (Received October 29, 1942.)

77. E. J. Purcell: *Flat space congruences of order one in $[n]$.*

By an $[n-k]$-congruence of order one in $[n]$, $(k<n)$, is meant an algebraic ∞^k-system of $[n-k]$'s in n-dimensional projective space such that one and only one $[n-k]$ of the system passes through an arbitrary point of $[n]$. This paper defines, classifies, and studies 2^{k-1} types of $[n-k]$-congruences of order one in $[n]$, for which the fundamental loci on a generic $[n-k]$ are all distinct. (Received November 2, 1942.)

78. W. H. Roever: *The axonometric method of representing the points of space on a plane.*

After stating that axonometry is one of the modern methods of descriptive geometry which is but little known in this country, the author describes his method of representing space upon the plane as follows:—Adjoin to the object to be represented its shadow on some fixed plane, such as the ground, and then project this augmented figure on the picture plane, thus obtaining thereon for each point P of the object its projection P^* and also the projection P'^* of its shadow. He thus obtains for a point P of space the two related picture-plane points P^* and P'^* which together furnish enough information to obtain therefrom definitely and uniquely the position of P in space. He also shows that this method yields the same result as that given by him in his monograph entitled *Fundamental theorems of orthographic axonometry and their value in picturization* (Washington University Studies, n.s., Science and Technology, no. 12, St. Louis, 1941). (Received November 19, 1942.)

79. Domina E. Spencer: *The tensor representation of the figures of Study's "Geometrie der Dynamen."*

In an earlier paper (*Geometric figures in affine space*, Bull. Amer. Math. Soc. abstract 48-9-281) the affine ancestors of the Study figures were investigated and the foundations were laid for the tensor representation of the figures themselves. The present paper continues the work by the introduction of a metric and the detailed study of the various Study figures. This systematic approach to the subject will make possible important applications which were obscured by the complexity of the original treatment of Study. (Received November 18, 1942.)

80. C. E. Springer: *Dual geodesics on a surface.*

In this paper a dual geodesic is defined to be a curve on a metric surface with the property that its ray-point corresponding to every point P on the curve lies on the line l_2 which is in Green's Relation R to the normal line l_1 to the surface at P. The differential equation of the dual geodesics is derived. The directions of Segrè are characterized as the directions in which the geodesics and dual geodesics coincide. Finally, the cubic curve, which is the locus of the ray-points of geodesics through a point of the surface, is studied. (Received October 27, 1942.)

81. S. M. Ulam: *On the length of curves, the surface area and the isoperimetric problem under a general Minkowski metric.* Preliminary report.

Given a metric in Euclidean space, defined through a symmetric convex unit gauge, one is led to the notion of length of curves through the usual process of polygonal approximation. The isoperimetric problem in the plane has a solution. A notion of surface area, invariant with respect to congruence under the given metric, is introduced. (Received November 23, 1942.)

82. J. E. Wilkins: *The first canonical pencil.*

It is the purpose of this paper to give several geometric definitions for a general canonical line of the first kind. Each such canonical line except the first axis of Čech may be defined in terms of the cusp-axes of the two families of hypergeodesics which are extremals of the integrals $\int \beta^n \gamma^{1-n} v'^{2-3n} du$, $\int \gamma^n \beta^{1-n} v'^{3n-1} du$, where n is constant, or as the cusp-axis of a cone of class three which is defined by means of the osculating planes of these hypergeodesics. If at each point of the surface is introduced the triple of directions $\overline{D}_k$ conjugate to the directions D_k considered by Bell (Duke Math. J.

vol. 5 (1939) pp. 784–788), the general canonical line as the line of intersection of the osculating planes of the projective geodesics in the directions $\overline{D}_k$ results. (Received October 2, 1942.)

83. Y. C. Wong: *Some Einstein spaces with conformally separable fundamental tensors.*

When the fundamental tensor $^*g_{\alpha\beta}$ of a Riemannian m-space is of the form $^*g_{ij}=[\rho(x^\alpha)]^{-2}g_{ij}(x^k)$, $^*g_{ip}=0$, $^*g_{pq}=[\sigma(x^\alpha)]^{-2}g_{pq}(x^r)$, $\alpha,\beta=1,\cdots,m$; $i,j,k=1,\cdots,n$; $p,q,r=n+1,\cdots,m$, it is said to be conformally separable; $^*g_{ij}$ and $^*g_{pq}$, with x^r and x^k, respectively, as parameters, are called its component tensors. The author studies in this paper the conformally separable tensor which is the fundamental tensor of an Einstein m-space and each of whose component tensors either is of dimension less than three or is a family of fundamental tensors of Einstein spaces. It is found that the constructions of such a conformally separable tensor is invariably reduced to the construction of the fundamental tensor g_{ij} of an Einstein n-space or a Riemannian 2-space for which the equation $y_{,ij}=-Ig_{ij}$ admits a non-constant solution for y, where the comma denotes covariant differentiation with respect to g_{ij} and I is an unspecified scalar. The author is content with this result, because the latter problem has already been considered in detail by H. W. Brinkmann in his study of Einstein spaces which are conformal to each other. (This paper will be published in the Trans. Amer. Math. Soc.) (Received October 2, 1942.)

Numerical Computation

84. H. E. Salzer and Abraham Hillman: *Exact values of the first 120 factorials.*

Due to their fundamental importance, the exact values of the first 120 factorials were computed and checked. 120! contains 199 digits. 100! agreed with Uhler's value (Proc. Nat. Acad. Sci. U.S.A. vol. 28 (1942) p. 61). When these values were compared with Potin's table of the first 50 factorials (*Formules et tables numériques*, p. 836) errors were found in Potin's values for 18!, 38!, 45!, and 50!. (Received November 11, 1942.)

Statistics and Probability

85. J. H. Curtiss: *A note on the theory of moment generating functions.*

The moment generating function (m.g.f.) of a variate X is defined as the mean value of exp (αX), the characteristic function (c.f.) as the mean value of exp (itX), where α and t are real. The purpose of this note is to place on record careful statements and proofs of the appropriate analogues for the m.g.f. of the well known uniqueness and limit theorems for the c.f. For example, Levy's continuity theorem assumes the following form: Let $F_n(x)$ and $G_n(\alpha)$ be, respectively, the d.f. and m.g.f. of a variate X_n. If $G_n(\alpha)$ exists for $|\alpha|<\alpha_1$ and for all $n\geqq n_0$, and if there exists a function $G(\alpha)$ defined for $|\alpha|\leqq\alpha_2<\alpha_1$, $\alpha_2>0$, such that $\lim_{n\to\infty}G_n(\alpha)=G(\alpha)$ uniformly, $|\alpha|\leqq\alpha_2$, then there exists a variate X with d.f. $F(x)$ such that $\lim_{n\to\infty}F_n(x)=F(x)$ uniformly in each finite interval of continuity of $F(x)$. The m.g.f. of X exists for $|\alpha|\leqq\alpha_2$ and is equal to $G(\alpha)$ in that interval. (Received October 9, 1942.)