ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

1. I. S. Cohen: *Some theorems on local rings.*

Let $\mathfrak{R}$ be a p-series ring (as defined by Krull, J. Reine Angew. Math. vol. 179 (1938) p. 205); let n be its dimension. It is shown that if $\mathfrak{a}$ is an ideal in $\mathfrak{R}$ having a basis of r elements and of dimension at most $n-r$, than $\mathfrak{a}$ is unmixed, of dimension $n-r$. This is a generalization of a well known theorem of Macaulay. If $\mathfrak{R}$ is complete and if $\mathfrak{S}$ is a p-series ring integrally dependent on $\mathfrak{R}$, then the rank of $\mathfrak{S}$ over $\mathfrak{R}$ is equal to the ramification order of $\mathfrak{S}$ with respect to $\mathfrak{R}$ (defined as the length of the primary ideal obtained by extending to $\mathfrak{S}$ the maximal ideal of $\mathfrak{R}$) multiplied by the degree of the residue field of $\mathfrak{S}$ over that of $\mathfrak{R}$. These results depend on the following theorem concerning the structure of a complete p-series ring: If the characteristics of $\mathfrak{R}$ and its residue field are the same, then $\mathfrak{R}$ is a power series ring over this field; if the characteristics are different, then under a simple additional hypothesis, $\mathfrak{R}$ is a power series ring over a complete discrete valuation ring. It is also shown that every complete local ring is a homomorphic map of a ring of one of these two types. (Received November 24, 1942.)

2. Franklin Haimo: *Periodic functions on algebraic systems.*

A single-valued function F over a group G is said to have a period p if $F(xpy)=F(xy)$ for every x and y in G. The periods of F form a normal subgroup of G. All single-valued functions over G with range in a class C are at least trivially periodic and are partitioned into classes each containing one and only one distinct homomorphism of G. Single-valued functions over a quasi-field H may have periods which are both additive and multiplicative. Such periods form a normal subgroup of the multiplicative group of H. Single-valued functions over lattices may have join-periods and meet-periods. Meet-periods form a join-ideal and dually; while non-constant single-valued functions over Boolean algebras have no members which are both join- and meet-periods. (Received November 23, 1942.)

3. P. R. Halmos: *On automorphisms of compact groups.* I.

If T is a continuous automorphism of a compact abelian group G then, because of the uniqueness of Haar measure, T is a measure preserving transformation. T is ergodic (in fact strongly mixing) if and only if the adjoint automorphism T^* of the character group G^* has no finite orbits. If an automorphism (such as T^*) of a discrete abelian group has no finite orbits then it has an infinite number of orbits. It follows easily that the spectral type of the unitary operator induced by an ergodic T on $L_2(G)$ depends only on the cardinal number of G^*. In case G is a finite-dimensional toral group (so that T is defined by a unimodular matrix) a simple condition on the coefficients of T is equivalent to ergodicity. This last remark puts in evidence many new and simple examples of analytic mixing transformations. A purely measure theoretic invariant of measure preserving transformations is proposed (along the lines of the characteristic equation theory of matrices); it is suggested that this invariant will serve to distinguish between transformations lumped together by the cruder spectral invariants. (Received November 17, 1942.)

4. Gerald Harrison: *The structure of algebraic moduls.*

A 1-1 correspondence between the submoduls of an algebraic modul of degree n and the set of all $n\times m$ matrices whose elements are rational integers may be set up if equivalence of matrices is defined in the usual manner. Nonsingular matrices whose elements are rational integers have multiplicative decompositions as the product of prime matrices, that is, matrices whose norms are primes. The number of such decompositions which a matrix has may be described in terms of the modular lattice which the set of all nonsingular matrices forms. This lattice has a direct product decomposition which is at the same time multiplicative, that is, if $a=(a_1, a_2, \cdots)$ and $b=(b_1, b_2, \cdots)$ then $a\cup b=(a_1\cup b_1, a_2\cup b_2, \cdots)$, $a\cap b=(a_1\cap b_1, a_2\cap b_2, \cdots)$, and $ab=(a_1b_1, a_2b_2, \cdots)$, where the multiplication referred to is that of algebraic moduls. (Received November 23, 1942.)

5. Olaf Helmer: *An extension of the elementary divisor theorem.*

A Prüfer ring is a domain of integrity in which all ideals possessing a finite basis are principal ideals. The elementary divisor theorem is known to hold in principal ideal rings as well as in rings with Euclidean algorithm; whether it will ever be possible to extend the theorem to Prüfer rings is questionable. In this paper it will be proved for a subclass of all Prüfer rings, to be called adequate rings, which more than comprise the class of rings hitherto known to be covered by the theorem. An adequate ring is defined as a Prüfer ring in which every nonzero element a can, for any element b, be split into two factors: $a=a_1\cdot a_2$, such that a_1 is a largest factor of a prime to b: $(a_1, b)=1$, largest in the sense that no factor a_3 of a_2 is prime to b: $(a_3, b)\neq 1$. The proof proceeds along familiar lines, after the following lemma has been proved: Let $M=(a_{ij})$, $(i=1, \cdots, r; j=1, \cdots, n)$ be a matrix with coefficients in an adequate ring R, let $1<r\leqq n$, rank $M=r$, and $(a_{11}, a_{12}, \cdots, a_{rn})=d$; then there exist $t_1, t_2, \cdots, t_{r-1}$ in R such that $(A_1, A_2, \cdots, A_n)=d$, where $A_j=a_{1j}t_1+\cdots+a_{r-1,j}t_{r-1}+a_{rj}$. (Received November 23, 1942.)

6. Jakob Levitzki: *On semi-nilpotent ideals.*

A ring T is called semi-nilpotent (in short: s.n.) if each subring of T which is generated by a finite subset of T is nilpotent; otherwise T is called semi-regular. Thus the sum $\overline{N}$ of all nilpotent right (left) ideals of any ring S is s.n. In a previous note the author has proved that each ring S possesses a uniquely determined maximal two sided s.n. ideal N which contains also all one-sided s.n. ideals of the ring. In the present note it is shown that if S is a ring with minimal condition for two-sided s.n. ideals, then N is nilpotent, and hence $N=\overline{N}$. Applied to semi-primary rings, this theorem yields certain generalizations of recent results due to C. Hopkins (Duke

Math. J. vol. 4 (1938) pp. 664–667), J. Levitzki (Compositio Math. vol. 7 (1939) pp. 214–222) and K. Asano (Proc. Imp. Acad. Tokyo vol. 15 (1939) pp. 288–291). (Received November 23, 1942.)

7. Jakob Levitski: *On the radical of a general ring.*

In the present note a definition of the radical is suggested which retains its significance also in the general case. This definition is based on the notion of s.n. (semi-nilpotent) ideal (its counterpart is the semi-regular ideal) which is defined as follows: R is a s.n. ideal if each finite set of elements in R generates a nilpotent ring. Each nilpotent ideal is s.n., and each s.n. ideal is a nil-ideal. The sum N of all right s.n. ideals is a s.n. two-sided ideal, which is called the radical. The radical N of any ring S contains all one-sided (and two-sided) s.n. ideals, and the radical of S/N is zero. These theorems lose their validity for general rings if the radical is defined (as has been hitherto the case) by replacing s.n. ideals either by nilpotent ideals (specialized radical) or nil-ideals (generalized radical). The radical contains the specialized radical, and is a subset of the generalized radical (in case the latter exists). The results described above are applied to primary rings. (Received October 5, 1942.)

8. Saunders MacLane and B. A. Lengyel: *Integral invariants of a tensor under the symmetry operations of the* 32 *crystal classes.* Preliminary report.

The theory of elastic and plastic deformations of an isotropic body makes use of the invariants of the stress tensor under the group of all orthogonal transformations. To extend this theory to crystalline media it is necessary to know the invariants of the finite groups of transformations characteristic of the crystal classes. The effect of the various symmetry operations on the components of a tensor was studied and an integrity basis was set up for the invariant polynomials for each of the 32 classes. The method of finding the integrity bases is elementary and consists of reducing the problem to that of the vector invariants of the symmetric and alternating groups, respectively. (Received November 24, 1942.)

9. C. J. Nesbitt and W. M. Scott: *Some remarks on algebras over an algebraically closed field.*

In this paper are examined some aspects of an algebra A which may have a radical and whose coefficient field is algebraically closed. A main concept is that of basic algebra. The basic algebra of A is a semi-primary subalgebra which for the algebra A plays a role in some respects analogous to that of division algebras in the theory of simple algebras. Related to the basic algebra are the Cartan basis systems and systems of elementary modules (W. M. Scott, Ann. of Math. (2) vol. 43 (1942) pp. 147–160). An algebra B is said to be similar to A if the basic algebra of B is isomorphic to that of A. Similar algebras form a class, and have corresponding representations. The commutator algebras of matrix representations of A are analyzed. The linear symmetric functions of A (abstract 44-11-452) are determined. The regular representations of A are written in terms of the elementary modules. (Received October 26, 1942.)

10. Ivan Niven: *An unsolved case of the Waring problem.*

The Dickson-Pillai-Vinogradow solution of Waring's problem (L. E. Dickson, Amer. J. Math. vol. 58 (1936) p. 535) leaves unsolved the case in which r, the remainder upon division of 3^n by 2^n, equals 2^n-q-2, q being the quotient. The present paper shows that in this case $g(n)$, the minimum number of positive or zero nth powers necessary to express every positive integer, has the "ideal" value 2^n+q-2. The method used is Dickson's, sharpened somewhat. (Received October 14, 1942.)

11. Ivan Niven: *The Pell equation in quadratic fields.*

In a previous report (*Quadratic Diophantine equations in the rational and quadratic fields*, Trans. Amer. Math. Soc. vol. 52 (1942) p. 2, Theorem 4) the author gave necessary and sufficient conditions that the equation $\xi^2-\gamma\eta^2=1$ have an infinite number of integral solutions ξ, η in any quadratic field, γ being a given integer of the field. It is shown here that if the equation has an infinitude of solutions, they can be obtained from one least solution (as in the case of the Pell equation in the rational field) if and only if γ is not a totally positive non-square integer of a real quadratic field. In case γ has this property it is shown that the equation has infinitely many solutions with ξ and η bounded. (Received October 16, 1942.)

12. Rufus Oldenburger: *The characteristic of a sum of quadratic forms.*

It is proved that the Loewy characteristic of a sum of quadratic forms is determined by properties of adjoint forms, and nonzero components in characteristic splittings of quadratic forms. (Received November 21, 1942.)

13. Edward Rosenthall: *Diophantine equations in arbitrary algebraic number fields.*

The complete solution in integers $XY=ZW$ in any algebraic integral domain is given by $eX=US$, $eY=VT$, $eZ=UT$, $eW=VS$ where e takes only the finite set of rational integral values, each equal to the norm of a representative ideal from each class, and U, S, V, T, are arbitrary integers of the field. Complete integer solutions of other multiplicative equations are deduced and in particular from one of these equations all sets of rational integers satisfying $ax^2+by^2=z^3$ are obtained in terms of arbitrary integral parameters subject to a congruential condition modulo e. (Received November 20, 1942.)

14. Edward Rosenthall: *Diophantine equations in biquadratic fields.*

Explicit complete rational integer solutions are obtained for some diophantine equations reducible in certain so-called special Dirichlet biquadratic fields. The equation $N(X)=N(Y)$ is solved completely in rational integers, where $N(A)$ denotes the normal of the biquadratic integer A; in particular this equation in $Ra[2^{1/2}+i]$ yields the complete rational integer solution of $x^4+y^4=N(s+it+2^{1/2}u+i2^{1/2}v)$ with at most one linear relationship connecting the coordinates s, t, u, v. A multiplicative equation in $Ra[3^{1/2}+i]$ gives the complete rational integer solution of $x^3+y^3=u^2+v^2$. The equation $x^3+y^3=u^3+v^3$ is also solved completely in Gaussian integers. (Received November 20, 1942.)

15. T. L. Wade: *Euclidean concomitants of the triangle.*

The results of the writer (Bull. Amer. Math. Soc. vol. 47 (1941) pp. 475–478 and vol. 48 (1942) pp. 589–601) which relate tensor algebra and invariant theory are used in the consideration of the elementary problem of the three-line configuration. (Received November 23, 1942.)

16. T. L. Wade: *On conjugate tensors.*

How a contravariant (skew-symmetric) tensor V of order $n-p$ may be associated with a covariant skew-symmetric tensor U of order p, in an n-dimensional coordinate system, is well known (see Veblen and von Neumann, *Geometry of complex domains*). This standard association holds only when U is skew-symmetric. The purpose of this note is to show how a contravariant tensor V, of defined order and symmetry, can be associated with the covariant tensor U, where U is of any type $[\alpha]$ of symmetry. (Received November 23, 1942.)

17. T. L. Wade: *On the factorization of rank tensors.*

Let $C^{(i)}_{(j)}=D^{(i)}_{(j)}+E^{(i)}_{(j)}$, where $D^{(i)}_{(j)}$ and $E^{(i)}_{(j)}$ are mutually orthogonal idempotent numerical tensors. Expressions for the contravariant and covariant factors of the rank tensor (see Amer. J. Math. vol. 64 (1942) pp. 725–752) of $C^{(i)}_{(j)}$ in terms of like factors of the rank tensors of $D^{(i)}_{(j)}$ and $E^{(i)}_{(j)}$ are established in this paper. (Received November 23, 1942.)

18. T. L. Wade and R. H. Bruck: *Types of symmetries.*

This paper considers some aspects of symmetries with tensorial significance which are believed not to have appeared in the literature. (Received November 23, 1942.)

19. André Weil: *Differentiation in algebraic number-fields.*

Analogies with function-fields have long ago led E. Noether and others to the conjecture that the theory of the different in number-fields can be built upon some arithmetical analogue of differentiation. This is now done, by defining a derivation modulo an ideal $\mathfrak{a}$ in a number-field as an operator D with the following properties: (a) D maps the ring $\mathfrak{o}$ of integers in the field into the ring $\mathfrak{o}/\mathfrak{a}$; (b) $D(\alpha+\beta)=D\alpha+D\beta$; (c) if $\bar{\alpha}$, $\bar{\beta}$ are the classes of α, β mod $\mathfrak{a}$, then $D(\alpha\beta)=\bar{\alpha}\cdot D\beta+\bar{\beta}\cdot D\alpha$; D is essential if there is α in $\mathfrak{o}$, such that $D\alpha$ is not a zero-divisor in $\mathfrak{o}/\mathfrak{a}$. The different is then the least common multiple of all ideals modulo which there exists an essential derivation. This is easily extended to the relative different, to p-adic fields, and so on. (Received November 9, 1942.)

20. Alexander Wundheiler: *An algebraic definition of affine space.*

A simple set of axioms for affine geometry based on one operation $C=hAB$, where h is a real number, A, B points and C the point collinear with A and B, and such that $CA/CB=h$, is given. There are essentially five axioms, only one of them involving more than two (namely, three) points. An "affine calculus," which permits the writing of every affine theorem as an implication between formulas, arises from the mentioned operation. (Received November 20, 1942.)

Analysis

21. R. P. Agnew: *Euler transformations.*

Let $E(r)$ denote the Euler transformation $\sigma_n=\sum_{k=0}^{n}C_{nk}r^k(1-r)^{n-k}s_k$ by means of