c_q is the number of classes of elements of order q in G, then not more than min c_q orthogonal squares can be constructed from G by the automorphism method. (Received September 30, 1942.)

337. Abraham Wald: *On a statistical problem arising in the classification of an individual in one of two groups.*

Let π_1 and π_2 be two p-variate normal populations which have a common covariance matrix. A sample of size N_i is drawn from the population π_i $(i=1, 2)$. Denote by $x_{i\alpha}$ the αth observation on the ith variate in π_1, and by $y_{i\beta}$ the βth observation on the ith variate in π_2. Let z_i $(i=1, \cdots, p)$ be a single observation on the ith variate drawn from a population π where it is known that π is equal either to π_1 or to π_2. The parameters of the populations π_1 and π_2 are assumed to be unknown. It is shown that for testing the hypothesis $\pi=\pi_1$ a proper critical region is given by $U\geqq d$ where $U=\sum\sum s^{ij}z_i(\bar{y}_j-\bar{x}_j)$, $\|s^{ij}\|=\|s_{ij}\|^{-1}$, $s_{ij}=[\sum_\alpha(x_{i\alpha}-\bar{x}_i)(x_{j\alpha}-\bar{x}_j)+\sum_\beta(y_{i\beta}-\bar{y}_i)(y_{j\beta}-\bar{y}_j)]/(N_1+N_2-2)$, $\bar{x}_i=(\sum_\alpha x_{i\alpha})/N_1$, $\bar{y}_i=(\sum_\beta y_{i\beta})/N_2$ and d is a constant. The large sample distribution of U is derived and it is shown that U is a simple function of three angles in the sample space whose exact joint sampling distribution is derived. (Received August 7, 1942.)

338. Jacob Wolfowitz: *On the consistency of a class of non-parametric statistics.*

Let X and Y be two stochastic variables about whose distribution nothing is known except that they are continuous and let it be required to test whether their distribution functions are the same. Let V be the observed sequence of zeros and ones constructed as described elsewhere (Wald and Wolfowitz, Annals of Mathematical Statistics, vol. 11 (1940), p. 148). Suppose that the statistic $S(V)$ used to test the hypothesis is of the form $S(V)=\sum\phi(l_j)$, where l_j is the length of the jth run and $\phi(x)$ a suitable function defined for all positive integral x. The notion of consistency, originated by Fisher for parametric problems, has already been extended to the non-parametric case (loc. cit., p. 153). The author now proves that, subject to reasonable conditions on $\phi(x)$ and statistically unimportant restrictions on the alternatives to the null hypothesis, statistics of the type $S(V)$ are consistent. In particular, a statistic discussed by the author (Annals of Mathematical Statistics, vol. 12 (1942)) and for which $\phi(x)=\log(x^x/x!)$ belongs to the class covered by the theorem. (Received August 7, 1942.)

Topology

339. O. G. Harrold: *A higher dimensional analogue of a theorem of plane topology.*

Since the carriers of a Vietoris cycle may have a dimensionality far removed from that of the cycle, it is of interest to determine a class of spaces for which the bounding cycles have membranes of dimensionality exceeding that of the cycle by unity. An example is known of an $1c^1$ carrying an essential 1-cycle which has a 1-dimensional carrier but bounds only on a 3-dimensional set. A similar example is constructed in the Euclidean space E_6. That such cannot happen in certain Euclidean spaces is indicated by the following theorem, which is a generalization of a known result for $n=0$. Let X be a compact $1c^n$ subset of E_{n+2}. Denote by F the frontier of X relative to E_{n+2}. There exists in X a compact subset X_0 which is $1c^n$ such that $X_0\supset F$ and dim $X_0\leqq n+1$. (Received August 4, 1942.)

340. F. B. Jones: *A characterization of the sphere.*

Let M denote a compact metric continuous curve such that (1) the sum of no two points separates M and (2) every simple closed curve in M separates M. Kline has raised the question as to whether M is a simple closed surface or not. The main result of this paper shows that it is if M contains no skew curve of type 1. (Received August 5, 1942.)

341. Saunders MacLane and Samuel Eilenberg: *Functions of groups as generalized tensors.*

Let Hom $\{G, H\}$ denote the group of all homomorphisms θ of the group G into the group H, and let α_1, β_1 be fixed homomorphisms of G into G_1, H into H_1, respectively. Then $\theta \in \text{Hom } \{G_1, H\}$ "induces" $\beta_1\theta\alpha_1 \in \text{Hom } \{G, H_1\}$, and the correspondence $\theta \rightarrow \beta_1\theta\alpha_1$ is itself a homomorphism $T(\alpha_1, \beta_1)$ of the first group (of θ's) into the second group Hom $\{G, H_1\}$. This function T has the property $T(\alpha_2\alpha_1, \beta_2\beta_1) = T(\alpha_1, \beta_2)T(\alpha_2, \beta_1)$. A function of two groups G, H, like the function "Hom," associated with such a function T of mappings of these groups, is called a tensor "covariant" in H, "contravariant" in G. The intuitive notion of a "natural isomorphism" between two groups can be defined exactly as an isomorphism between tensor functions of groups. This paper extends the same concepts from groups to topological spaces, to rings, to complexes, and to other types of mathematical systems. A number of general properties of tensors and their isomorphisms are established. (Received August 3, 1942.)

342. R. L. Moore: *Concerning a continuum and its boundary.*

If H, K and T are subsets of the continuum M then T is said to shield H from K in M provided it contains no point of K but intersects every connected subset of M which intersects both H and K. Among other things, it is shown that if, in a metric space, A and B are points of β, the boundary of the compact continuum M, and for each component x of $M-\beta$ there is a continuum β_x lying in β and shielding $A+B$ from x then A and B belong to the same component of β. (Received September 10, 1942.)

343. R. L. Moore: *Concerning intersecting continua.*

One of the results obtained is the theorem that if, in a metric space, β is the boundary of the compact continuum M and K is a compact continuum intersecting M and, for every component D of $M-\beta$ and component E of the complement of $\bar{D}$, the boundary of E is a connected subset of β, then $M\cdot K$ is a subset of a component of $M\cdot K+\beta$. (Received September 10, 1942.)

344. C. N. Reynolds: *A calculus of finite topological differences with application to the four color problem.*

A topological derivative of any function with respect to a well defined transformation of the neighborhood of an arbitrary boundary of a map is defined and is so applied to the four color problem as to reduce it to an equivalent problem in the theory of functions of a real variable. (Received August 4, 1942.)

345. A. D. Wallace: *A characterization of unicoherence.*

Let S be a Peano space. If $S=A+B+C$ is a decomposition into continua such that AB, BC, $CA \neq 0$ then assuming S to be unicoherent it is readily seen that $ABC \neq 0$. For $S=A+(B+C)$ so that, since $B+C$ is a continuum, so also is $A(B+C)=AB+AC$. The summands in the right member being non-void and closed it follows that they must intersect. The converse of this is also true and the proof is contained implicitly in the proof of Theorem 1 of *Monotone coverings and monotone transformations*, Duke Mathematical Journal, vol. 6 (1940), p. 32. (Received September 15, 1942.)

346. G. T. Whyburn: *Homotopy classes of mappings into the circle.*

In this paper it is shown primarily that any continuous mapping of a locally connected continuum X into a circle S is homotopic to a mapping f which admits factorization into the form $f=f_2f_1$, where f_1 is monotone and f_2 is interior and light (thus f is quasi-monotone). Used in the proof of this main theorem are auxiliary results to the effect that (1) any continuous mapping f of X into S is interior at every $x \in X$ which is a component of $f^{-1}f(x)$ and which is not contained in arbitrarily small neighborhoods U with $F(U) \subset f^{-1}(y)$ for some $y \in S-f(x)$; (2) a light mapping f of X into S is interior if and only if, for any $y \in S$, every component of $X-f^{-1}(y)$ maps onto $S-y$; (3) any non-alternating mapping f of X into S factors into the form $f=f_2f_1$, where f_1 is monotone and f_2 is non-alternating, interior and light; thus any non-alternating light mapping of X into S is interior; (4) any continuous mapping f of X into S such that, for any $y \in S$, each component of $X-f^{-1}(Y)$ maps onto $S-y$ is quasi-monotone. (Received August 5, 1942.)

347. J. W. T. Youngs: *A note on separation axioms and their application in the theory of a locally connected topological space.*

This note is in the nature of a sequel to a paper by G. E. Albert and the author (*The structure of locally connected topological spaces*, Transactions of this Society, vol. 51 (1942), pp. 637–654). A new separation axiom is introduced. T: If $x \cdot y=0$, then $\bar{x} \cdot \bar{y}=0$. In strength this lies between the T_0 and T_1 axioms. Using the terminology of the paper mentioned above it is shown that the class of locally connected T-spaces is hereditary. (Received August 3, 1942.)