but have the advantage of being all distinct in a period, thus leading to a more incisive classification than is possible with the digits. (Received August 4, 1942.)

306. Gordon Pall: *The distribution of integers represented by binary quadratic forms.*

The formula due to R. D. James (American Journal of Mathematics, vol. 60 (1938), pp. 737–744) for the number of integers m prime to d represented by binary quadratic forms of discriminant d, is here freed of the restriction that m be prime to d. (Received August 3, 1942.)

307. Gordon Pall: *The weight of an n-ary genus of quadratic forms.*

The formula for the weight of a genus of integral positive quadratic forms in n variables is obtained in an explicit, useful form. The calculation of the factor for $p=2$ is greatly facilitated by the use of a much simplified system of invariants. There are numerous applications. (Received August 3, 1942.)

308. J. F. Ritt: *Bezout's theorem and algebraic differential equations.*

The intersection of the general solutions of two differential polynomials in two unknowns is examined with respect to the numbers of arbitrary constants on which its various irreducible components can depend. (Received August 6, 1942.)

309. Ernst Snapper: *The resultant of a linear set.*

The m-dimensional vector space $\bar{V}_m$ consists of vectors having, as components, m polynomials of the ring $P[y_1 \cdots y_n]$ where P is a field. The linear subsets of $\bar{V}_m$ are generated by the columns of $m \times s$ matrices with elements in $P[y_1 \cdots y_n]$. The ideal theory of $P[y_1 \cdots y_n]$, given by Hentzelt and Noether (Mathematische Annalen vol. 88 (1922), pp. 53–79), holds for these linear sets. By a linear, invertable transformation of the variables $y_1, \cdots, y_n$, which involves adjoining new variables γ_{ij} to P, the linear subsets of $\bar{V}_m$ become "transformed" linear sets of the vector space V_m over $P(\gamma)[x_1 \cdots x_n]$. Every transformed linear set L of V_m has a resultant $\rho \in P(\gamma)[x_1 \cdots x_n]$, which vanishes for, and only for, the zeros of the ideal $L/\bar{L}$. (See Snapper, Transactions of this Society, vol. 52 (1942), pp. 258–259 for the definitions of $\bar{L}$ and $L/\bar{L}$.) If $L_1 \subseteq L_2$, then $L_1 = L_2$, if and only if they have equal ranks and resultants. This gives a criterion for the existence of a polynomial solution of simultaneous linear equations with polynomials as coefficients. For $n=1$, the resultant becomes the highest dimensional determinantal factor of L. (Received September 29, 1942.)

Analysis

310. M. A. Basoco: *On the Fourier developments of a certain class of theta quotients.*

This paper is concerned with the functions $\phi_\alpha^k(z) \equiv \{\vartheta_\alpha'(z, q)/\vartheta(_\alpha z, q)\}^k$ $(\alpha=0, 1, 2, 3)$ where $\vartheta_\alpha(z, q)$ is a Jacobi theta function and k is a positive integer. The Fourier expansions of these functions are investigated and their arithmetical form is obtained for the cases $k=1, 2, 3$. Using these results and certain simple identities, there is obtained using the method of paraphrase (E. T. Bell, Transactions of this Society, vol. 22 (1921), pp. 1–30 and 198–219; *Algebraic Arithmetic*, American Mathematical Society Colloquium Publications, vol. 7, 1927, chap. 3), a series of theorems on numerical functions. Among these there appears a theorem due to Liouville (Journal de Mathématiques Pures et Appliquées, (2), vol. 3 (1858), p. 247), which has only recently been proved by elementary means by J. V. Uspensky (*Elementary Number Theory*, New York, 1939, p. 462). An application of these theorems leading to relationships between infinite series of the Lambert type is indicated. This paper will appear shortly in this Bulletin. (Received September 26, 1942.)

311. L. W. Cohen: *Integration on hypersurfaces.*

An invariant measure is developed for m-dimensional surfaces Σ_m which yields proofs of the theorems of Gauss and Stokes in terms of Lebesgue integrals. A surface Σ_m is a complex of m-cells $R_x^m = X(R_u^m)$ where R_u^m is an m-dimensional interval and $X = X(U)$ is a 1-1 mapping of R_u^m on an arbitrary set. To each $X \subset R_x^m$ are assigned numbers $x_1, \cdots, x_m$, $n \geqq m$, which are almost everywhere differentiable functions of the coordinates of $U \subset R_u^m$. The measurable sets $E_x \subset R_x^m$ are the images of the measurable sets $E_u \subset R_u^m$. The measure of E_x is the integral of $D_m^n(U)$ over E_u where $(D_m^n(U))^2$ is the Gram function of the $\partial x_i / \partial u_j$. The measure on Σ_m is determined by the measure on its several cells. For the Gauss theorem, Σ_m need not be orientable and may have singularities on its boundary while the functions are summable on the boundary and have finite summable derivatives on Σ_m. The Stokes theorem states the invariance of a certain integral on a family of homologous surfaces. (Received August 12, 1942.)

312. Willy Feller: *On some geometric inequalities.*

The following problem was originally formulated by R. Salem and D. C. Spencer in connection with a number-theoretical investigation. Consider a domain Γ contained in the unit sphere of R_n; suppose that the intersection of Γ with any straight line has a measure not exceeding a fixed constant $\delta < 1$. The problem is to determine the maximum measure of Γ. In the present paper this and a more general analytical problem are solved. At the same time an extremely simple proof is given for the well known inequality for the measure of convex sets with the above property (due to Bieberbach and Kubota). (Received August 3, 1942.)

313. Einar Hille and Gabor Szegö: *On the complex zeros of the Bessel functions.*

This paper contains a new proof of the theorem of A. Hurwitz according to which $z^{\beta/2} J_{-\beta}(2z^{1/2})$ has exactly $[\beta]$ non-positive zeros when $\beta \geqq 0$. The proof is based upon the fact that $n^{\beta} L_n^{(-\beta)}(z/n)$ tends to the function in question and the number and position of the non-positive zeros of Laguerre polynomials of negative order are easily discussed. (Received August 3, 1942.)

314. Abraham Hillman and H. E. Salzer: *Complex roots of* $\sin z = z$.

The first ten nonzero roots of $\sin z = z$ in the first quadrant were computed to seven decimal places. Obviously the roots are symmetrically situated in the four quadrants. The imaginary part, y, of the nth root satisfies the equation $x_1(y) - x_2(y) = 0$, where $x_1 = \coth y(\sinh^2 y - y^2)^{1/2}$ and $x_2 = 2n\pi + \text{arc} \cos (y/\sinh y)$. The y of the root was found by calculating $x_1 - x_2$ in the neighborhood of G. H. Hardy's approximation $y = \log (4n+1)\pi$ and then using inverse interpolation. The real part, x, was then taken as $x_2(y)$. The x and y were checked by the relation $x = x_1(y)$. (Received September 30, 1942.)

315. Ervand Kogbetliantz: *Inequality for definite integrals.* Preliminary report.

It is shown that the inequality for definite integrals $\int_a^b f(x)g(x)dx$ recently established by Shohat admits of a simple geometrical interpretation. This permits the extension of the inequality to the case where first n moments of $f(x)$ and $g(x)$ in $[a, b]$ vanish. (Received September 30, 1942.)

316. D. M. Krabill: *On extension of Wronskian matrices.*

The Wronskian matrix of order $n-1$ for a given set of solutions of a differential equation of the type (H): $y^{(n)}+f_1(x)y^{(n-1)}+\cdots+f_n(x)y=0$ with the $f_i(x)$ real and continuous in an interval J, has constant rank r in J and every column is a linear combination with constant coefficients of some r of the columns. The author gives a sufficient condition that an arbitrary set of functions having suitable class properties be solutions of an equation of type (H). The principal theorem states: If $k\leqq s\leqq t$ and the Wronskian matrix is of order s, $M_s(u_1, \cdots, u_k)$, for k arbitrary functions of class $C^{(t)}$ in J has constant rank k, there exists a function u_{k+1} of class $C^{(t)}$ with the extended matrix $M_s(u_1, \cdots, u_{k+1})$ of rank $k+1$ throughout J. A theorem of Curtiss (Mathematische Annalen, vol. 65 (1908), pp. 282–298, Theorem 4) on the zeros of the Wronskian determinant $W(u_1, \cdots, u_k)$ is utilized. From the extension property of Wronskian matrices is obtained the result: If the Wronskian matrix of order $n-1$ for k functions of class $C^{(t)}$ $(k\leqq n\leqq t)$ in an interval J has constant rank k, then the u's are linearly independent solutions of an equation of type (H) with the $f_i(x)$ of class $C^{(t-n)}$ in J. (Received August 3, 1942.)

317. Lee Lorch: *Lebesgue constants for Borel summability of Fourier series.*

Let $L_B(u)$ denote the uth Lebesgue constant for Borel summability of Fourier series. C. N. Moore (Proceedings of the National Academy of Sciences, vol. 11 (1925)) remarked that $L_B(u)$ becomes logarithmically infinite with u. Here it is shown that $L_B(u)=(2/\pi^2)\log u-(2/\pi^2)\log(\pi^2/2)-2C/\pi^2-(2/\pi^2)\int_0^\pi\psi(t/\pi)\sin t\,dt+O(u^{-1/2})$ as u becomes infinite, where $\psi(t)=\Gamma'(t)/\Gamma(t)$ and C is the Euler-Mascheroni constant. (Received August 3, 1942.)

318. Szolem Mandelbrojt: *Quasi-analyticity and properties of flatness of entire functions.*

The author proves a general theorem which may be regarded as a theorem concerning quasi-analyticity as well as the behaviour of entire functions on a set of points. This theorem relates an integrable function to an entire function. Each time that an entire function is chosen there results a theorem on quasi-analyticity, and when a suitable integrable function is given a theorem concerning entire functions follows. This paper will appear in the Duke Mathematical Journal. (Received September 14, 1942.)

319. W. T. Reid: *Some results on the growth of solutions of differential systems.*

In this paper elementary differential and integral inequalities are used to prove various results on the growth of solutions of differential systems, both linear and nonlinear, and involving functions of a real independent variable. In particular, the methods herein employed afford very brief proofs and improved forms of many of the results recently established by Trjitzinsky (Transactions of this Society, vol. 50 (1941), pp. 252–294). The paper also contains applications of an improved form of a theorem on matrix transformations due to Perron (Mathematische Zeitschrift, vol. 32 (1930), pp. 465–473). (Received September 7, 1942.)

320. R. M. Robinson: *Bounded analytic functions.*

(1) Suppose $g(z)$ is regular and satisfies $|g(z)| \leqq 1$ for $|z| < 1$. The relations between $g(0)$, z_0, $g(z_0)$, and $g'(z_0)$ are studied. For example, it is shown that for given values of $g(0)$ and z_0, the possible values of $g'(z_0)$ fill a closed convex set, which is bounded by a circle, a limaçon, or partly by one and partly by the other. The relations between the absolute values of the four quantities mentioned are also considered. (2) Suppose $f(z)$ satisfies the same conditions as $g(z)$, and also $f(0)=0$. The relations between $f'(0)$, z_0, $f(z_0)$, $f'(z_0)$ are studied. The relations between their absolute values are also considered. For example, the sharp lower and upper bounds for $|f'(z_0)|$ in terms of $|f'(0)|$ and $|z_0|$ are found. The lower bound is given by $|f'(z_0)| \geqq [|f'(0)| \cdot (1+|z_0|^2) - 2|z_0|]/(1-|f'(0)z_0|)^2$ provided the right side is positive, and is 0 otherwise; the upper bound is given by four formulas, valid for different values of $|f'(0)|$ and $|z_0|$. The relations between $|f'(0)|$, $|z_0|$, $|f(z_0)|$, and $|f'(z_0)|$, for univalent functions, were studied by the author in an earlier paper (Transactions of this Society, vol. 52 (1942), pp. 426–449). (Received September 16, 1942.)

321. P. C. Rosenbloom: *The zeros of the partial sums of power series representing entire functions.*

Let $f(z)$ be an entire function of order $0 < \rho \leqq \infty$. Consider the zeros of the nth partial sum $s_n(z)$ for large n. If $\rho = \infty$, it is proved that there are a sequence $\{n_i\}$ and a sequence of radii $\rho_i \to \infty$ such that if ϵ, $\delta > 0$, then there is an $i_0 = i_0(\epsilon, \delta, f)$ for which the number of zeros of $s_{n_i}(z)$ in the ring $\rho_{n_i}(1-\delta) \leqq |z| \leqq \rho_{n_i}(1+\delta)$ is $n_i(1-\eta)$ with $0 \leqq \eta < \epsilon$. The arguments of the zeros of $s_{n_i}(z)$ are equally distributed in the interval $0 \leqq \theta \leqq 2\pi$. If R_{n_i} is the maximum modulus of the zeros of $s_{n_i}(z)$, then lim sup $R_{n_i}/\rho_{n_i} \leqq 2$. The radius ρ_{n_i} can be chosen as $|a_{n_i}|^{-1/n_i}$ where a_{n_i} is the nth coefficient in the Taylor series for $f(z)$. These results constitute a sharpening of some theorems of Carlson (Comptes Rendus de l'Académie des Sciences, Paris, 1924), whose proofs have never been published (so far as we know). These results bear a striking analogy to the results of Jentzsch (*Untersuchungen zur Theorie der Folgen analytischen Funktionen*, Dissertation, Berlin, 1914) and Szegö (*Über die Nullstellen der Polynomen, die in einem Kreise gleichmässig konvergieren*, Sitzungsberichte der Berliner Mathematische Gesellschaft, vol. 21 (1922)) for power series with a finite radius of convergence. For functions of finite order, lim sup $R_{n_i}/\rho_{n_i} \leqq 2 \exp (1/\rho)$ for a certain sequence $\{n_i\}$. The paper also includes a detailed investigation of certain functions of finite order such as the error function, with results similar to Szegö's, for the exponential function (*Über eine Eigenschaft der Exponentialreihe*, Sitzungsberichte der Berliner Mathematische Gesellschaft, vol. 23 (1924)). (Received August 31, 1942.)

322. Max Shiffman: *Unstable extremal surfaces for double integral problems in the calculus of variations.*

Theories of the absolute minimum for double integral problems have been established in recent years. In this paper the author develops a theory of unstable extremal surfaces, proving the general Morse relations. The integrals to be made stationary have the form $\iint\{f(X, Y, Z)+k(X^2+Y^2+Z^2)^{1/2}\}\,dudv$ where X, Y, Z are the Jacobians $(y_u, z_u; y_v, z_v)$, and so forth, k is a positive constant, and f satisfies the following four conditions: (1) f is positively homogeneous in X, Y, Z; (2) $\epsilon=f(X, Y, Z)-\{Xf_{X_0}+Yf_{Y_0}+Zf_{Z_0}\}\geqq 0$; (3) $f(X, Y, Z)\geqq m>0$ for $X^2+Y^2+Z^2=1$; and (4) $f(X, Y, Z)<k$ for $X^2+Y^2+Z^2=1$. The first three are usual conditions to be expected; the main restriction is (4) which asserts that the integral is dominantly an area integral. The problem is to find extremal surfaces bounded by a given rectifiable curve. The integral is replaced by $\iint f(X, Y, Z)dudv+kD[x]$ where $D[x]$ is the Dirichlet integral, and the extremal surfaces found are given in isometric representation. (Received August 5, 1942.)

323. J. A. Shohat: *An inequality for definite integrals*. Preliminary report.

Starting with a simple case of the Schwarz inequality, another inequality is derived in a very elementary manner. This inequality seems to be new and may be considered an improvement over the Schwarz inequality. In fact, it is shown that the new inequality yields better results than the Schwarz inequality (it reduces to the latter, in some special cases).—Applications are indicated to orthogonal functions and to polynomials. (Received September 17, 1942.)

Applied Mathematics

324. Lipman Bers and Abe Gelbart: *On solutions of the differential equations of gas dynamics*. I.

By a transformation due to Chaplygin and Busemann the equations of the steady two-dimensional irrotational flow of a gas can be reduced, in the subsonic case, to the form (1): $u_x=\tau(y)v_y$, $u_y=-\tau(y)v_x$, where τ is a given function. Equations of the same form occur in other branches of mechanics of continua. The class of complex-valued functions $f(x+iy)=u+iv$, where u and v satisfy (1) is shown to have many properties in common with analytic functions. Two processes, one the inverse of the other, are introduced: τ-differentiation and τ-integration. The τ-derivative and the τ-integral of f belong to the class. By τ-integrating a complex constant, a_n, n times a function $a_n\cdot Z\cdot{}^n(z)$, of the class is obtained. From these "powers," " polynomials" and "power series" are formed. A "polynomial" of the nth degree possesses n zeros. Any function, f, of the class can be developed in a unique manner in a power series, the "coefficients" a_n, being given by the nth τ-derivatives of f. Domains of convergence, mapping properties, and particular functions are considered. Analogous methods are applied to more general types of equations; in particular to the system $u_x=\tau_1(y)v_y$, $u_y=-\tau_2(y)v_x$ which describes the subsonic as well as supersonic flow. (Received August 27, 1942.)

325. Garrett Birkhoff: *A reversibility paradox in hydrodynamics*.

It is shown that every known differential equation and general boundary condition describing compressible non-viscous flow is reversible, in the sense that it is compatible with reversal of the direction of flow without change in pressure. This is even true in the presence of a conservative force-field. It follows that, contrary to a widespread impression, any theory of non-viscous flow (airfoil theory, resistance to projectiles, or resistance to surface craft in water) which purports to be based entirely on