trowski, Bourion, and others is here generalized and unified by the concept of exact harmonic majorant of a sequence of analytic functions. If the function $V(z)$ is harmonic in a region R of the z-plane, if the functions $F_n(z)$ are locally single-valued and analytic in R except for branch points, and if $|F_n(z)|$ is single-valued in R, then $V(z)$ is said to be an exact harmonic majorant of the sequence $F_n(z)$ in R provided one has $\limsup_{n\to\infty}[\max |F_n(z)|, z \text{ on } Q] = [\max e^{V(z)}, z \text{ on } Q]$ for every continuum Q (not a single point) in R. Applications of this concept involve degree of convergence and properties of the zeros of functions, and include maximal sequences of polynomials and of other rational functions, and many other sequences of analytic functions. (Received March 16, 1942.)

197. M. S. Webster: *A convergence theorem for certain Lagrange interpolation polynomials.*

A convergence theorem for a sequence of Lagrange interpolation polynomials based on the zeros of a sequence of certain Jacobi polynomials is proved. The method and result are similar to a theorem of Grünwald (this Bulletin, vol. 47, (1941), pp. 271–275). (Received March 19, 1942.)

198. Hermann Weyl: *Solution of the simplest boundary-layer problems in hydrodynamics.*

For some simple configurations the hydrodynamic boundary-layer problem can be reduced to a non-linear ordinary differential equation of third order involving a parameter λ. For $\lambda=0$ and $1/2$, solution may be obtained by a rapidly converging process of alternating successive approximations. The general case is attacked by a suitable adaptation of the method of fixed points of transformations in functional spaces. (Received February 28, 1942.)

199. František Wolf: *On the limits of harmonic and analytic functions along radii which form a set of positive measure.*

If $u(r, \theta) = \log|f(re^{i\theta})|$ and $f(z)$ is analytic in the unit circle $r<1$, $u(r, \theta) \leqq M/(1-r)^n$ for any M and n, and $\limsup_{r\to 1} u(r, \theta) \leqq 0$ for $\theta \subset E$, $|E|>0$, then $\limsup u(r, \theta) \leqq 0$ in any sector at almost all points of E. Hence if $u(r, \theta)$ is harmonic and satisfies the conditions of the theorem, then $u(r, \theta)$ and its conjugate $v(r, \theta)$ have finite limits in any sector at almost all points of E. This follows from above by the well known results of Privaloff (Recueil Mathématique de Moscou, vol. 91 (1923), p. 232) and Fatou. Another corollary is: If $f(z)$ is analytic in $|z|<1$, $|f(z)| \leqq \exp [M/(1-r)^n]$, and $\lim_{r\to 1} f(re^{i\theta}) = 0$ for $\theta \subset E$, $|E|>0$, then $f(z) \equiv 0$. (Received March 20, 1942.)

Applied Mathematics

200. Stefan Bergman: *Determination of pressure in the two-dimensional flow of an incompressible perfect fluid.*

The author considers a flow of an incompressible perfect fluid around a wing profile. The pressure distribution is determined by the function $W(z)$ which maps the exterior $\mathfrak{E}$ of the wing profile onto the exterior $\mathfrak{R}$ of a circle. Of particular interest is the evaluation of $W(z)$ on the boundary in the neighborhood of the vertex O. Let boundary in the neighborhood of O be formed by two circular arcs CO and BO which make an angle α at O. Let O and D be the intersections of the circles on which the arcs CO and BO lie. Suppose that arcs CD and BD lie inside of the profile (Hypothesis

H). Then a linear transformation followed by one of the form $\zeta^* = \zeta^{\pi/(2\pi-\alpha)}$ will map $\mathfrak{C}$ into a domain $\mathfrak{T}$ which lies in the upper half-plane, and the arcs OA and OC will become segments O_2A_2 and O_2C_2 of the real axis. Using orthogonal functions (see Bergman, Publication of Brown University, 1941, p. 118), the author determines the function $w(\zeta^*)$ mapping $\mathfrak{T}+\mathfrak{T}^*$ into the unit circle. $\mathfrak{T}^*$ is the domain obtained from $\mathfrak{T}$ by reflection on the real axis. Then $(1/2)[w(\zeta^*)+(w(\zeta^*))^{-1}]^{-1}$ maps $\mathfrak{T}$ into $\mathfrak{R}$. If H is not fulfilled, follow the linear transformation by one which maps the triangle $O_1A_1C_1$ of the ζ-plane into the upper half-plane. (Received February 21, 1942.)

201. Stefan Bergman: *Three-dimensional flow of a perfect incompressible fluid and its singularities.*

The author considers vectors $\mathfrak{S}(\mathfrak{X})$, $\mathfrak{X}=(x_1, x_2, x_3)$. The components of $\mathfrak{S}$ are harmonic functions with an algebraic singularity described in the author's papers (Mathematische Zeitschrift, vol. 24 (1925–1926), p. 655 and Mathematische Annalen, vol. 99 (1928), p. 645). At every point outside of the singularity, curl $\mathfrak{S}=0$, div $\mathfrak{S}=0$ holds. Let $\theta(\mathfrak{X})$ be a potential function such that grad $\theta(\mathfrak{X})=\mathfrak{S}(\mathfrak{X})$. The author defines a curve $l^1(\zeta)$ in the $x_1x_2x_3$-space for every $\mathfrak{S}(\mathfrak{X})$ and every complex number ζ. Let q^1 be a closed curve in the ζ-plane. $L^2(q^1)$ is the logical sum of $l^1(\zeta)$, $\zeta\neq q^1$, so that to every point $\mathfrak{X}$ of $L^2(q^1)$ corresponds a $\zeta=\zeta(\mathfrak{X})$. [The singularity line of $\mathfrak{S}(\mathfrak{X})$ lies on $L^2(q^1)$.] Finally the author defines for $\mathfrak{S}(\mathfrak{X})$ the residue functions $R_\nu(\zeta)$. Let i^1 be a closed curve in the schlicht $x_1x_2x_3$-space. i^1 is open in the multiply-covered space M^3 in which $\mathfrak{S}(\mathfrak{X})$ is univalent. Let $\mathfrak{E}_1$ and $\mathfrak{E}_2$ be both end points of i^1 in M^3. Then $\int_{i^1}\mathfrak{S}(\mathfrak{X})\cdot d\mathfrak{X}+\sum[\theta_k(\mathfrak{E}_2)-\theta_k(\mathfrak{X}_k^*)]+\sum[\theta_n(\mathfrak{X}_n^*)-\theta_n(\mathfrak{E}_1)]=\sum\int_{\zeta(\mathfrak{X}_\mu^*)}^{\zeta(\mathfrak{X}_{\mu+1}^*)}R_\mu(\zeta)d\zeta$. Here $\theta_m(\mathfrak{X})$ are functions connected with $\theta(\mathfrak{X})$, and $\mathfrak{X}_m^*$ intersections of i^1 with $L^2(q^1)$. An analogous relation holds for a vector $\mathfrak{H}$, $\mathfrak{H}=\mathfrak{H}_1+\sum_{\nu=1}^{N}\mathfrak{S}_\nu$, where $\mathfrak{H}_1$ is a regular vector and $\mathfrak{S}_\nu$ the above described vectors. (Received March 6, 1942.)

202. Hilda P. Geiringer: *On the numerical solution of linear problems by group iteration.*

The so called Ph. Seidel iteration method for solving systems of linear equations converges towards their solution for all systems originated from a minimum problem; for example, in the following groups of problems: (1) statistical (least square) problems, (2) problems of mechanics, for example, statically determined or indeterminate frameworks, (3) elliptic boundary value problems. For the second group the method reduces the solution of an n-fold indeterminate system to the successive solution of simply indeterminate systems. (a) Each of these simply indeterminate systems can be solved in whatever way seems appropriate, not necessarily through the respective Maxwell equation. (b) Introduce "group-iteration" (see H. Geiringer, *Zur Praxis der Lösung linearer Gleichungen in der Statik*, Zeitschrift für Angewandte Mathematik und Mechanik, 1928, and R. von Mises and H. Geiringer, *Praktische Verfähren der Gleichungsauflösung*, ibid., 1929); that is, instead of reducing to simply indeterminate problems, use as intermediate steps r-fold indeterminate systems whose solutions may be found by any suitable combination of equations. Thus liberty of arrangement results and the convergence is accelerated.—It seems that this procedure contains the essentials of R. V. Southwell's "relaxation method" successfully applied by him since 1933 to many mechanical problems and recently presented in a comprehensive work. The group iteration method combined with the use of computing machines to solve the small groups of equations leads to very good results. (Received February 9, 1942.)

203. A. M. Gelbart: *Bounds for pressure in a two-dimensional flow of an incompressible perfect fluid.*

It is known that the problem of the pressure distribution along a wing in the case of a two-dimensional flow of an incompressible perfect fluid can be reduced to the problem of determining the function which maps the exterior domain into the exterior of a circle. This paper deals with some properties of the function in the neighborhood of the angle of the wing. Bergman treats this problem by employing orthogonal functions and certain special transformations. (See abstract 48-5-200.) Using this approach, some inequalities previously obtained by the author for the coefficients of the mapping function, and a minimum integral, inequalities for the velocity in the neighborhood of the angle are obtained which depend only upon a suitable domain in which the boundary of the profile lies. (Received March 7, 1942.)

204. W. A. Mersman: *Heat conduction in a finite composite solid.*

The problem of one-dimensional heat conduction in a composite wall has been solved by Churchill (Duke Mathematical Journal, vol. 2 (1936), pp. 405–414, and Mathematische Annalen, vol. 115 (1938), pp. 720–739), the solution being presented in the form of a series which converges rapidly for large time values. The present paper furnishes a transformation of Churchill's solution in the form of a series which converges rapidly for small time values. This is done by expanding the Laplace transform of the solution as a geometric series and inverting term-by-term, instead of applying the Mittag-Leffler theorem and the inversion theorems of Doetsch and Churchill. (Received February 19, 1942.)

205. W. A. Mersman: *Heat conduction in an infinite composite solid with an interface resistance.*

The problem of one-dimensional heat conduction in a doubly infinite composite solid with an interface resistance is solved by the Laplace transformation method. The interface conditions are: (1) the product of conductivity and temperature gradient is continuous across the interface; (2) the temperature discontinuity across the interface is proportional to the product in (1) above, the factor of proportionality being a constant. (Received February 9, 1942.)

Geometry

206. Herbert Busemann: *Spaces with convex spheres.*

In a metric space a continuous curve which is locally isometric with a euclidean straight line will be called a geodesic. The paper considers a finitely compact metric space in which there is exactly one geodesic through any two different points. With an obvious definition of a tangent of a sphere a sphere is called convex if no tangent of the sphere contains interior points of that sphere. Assume that all spheres are convex and that the space has dimension greater than or equal to 3. The space is congruent to an elliptic space, as soon as at least one geodesic is closed. If all geodesics are open, convexity of the spheres as defined above coincides with the usual idea that a segment whose end points are in a sphere lies completely in the sphere, but the space is not necessarily flat. However if the parallel axiom (properly formulated) holds, the space is flat and its metric is Minkowskian. (Received March 20, 1942.)