ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

102. A. A. Albert: *Non-associative algebras. II. New simple algebras.*

It is first shown that non-associative algebras A with a unity quantity e and order n have the same properties for decomposition in direct sums as do associative algebras. Let G be any multiplicative group of order n of non-singular linear transformations S on A such that $eS=e$, H be a subset containing I, g be a set of $g_{S,T}$ in A defined for every S and T of G and which are not zero divisors. Then construct the crossed extension $E=(A, G, H, g)$. It is an algebra of order nm with e as unity quantity. For separable algebras A conditions are given that E be simple and central simple. It is always simple (central simple) when A is and when $H=[I]$. Then an iterative process results for extending ordinary crossed products of order r^2 to central simple algebras of order r^t which are necessarily non-associative. Every central simple algebra of order n may be extended by the use of an arbitrary permutation group on n letters and a class of permutation algebras is obtained. Finally, a list of fundamental unsolved problems is given. (Received January 7, 1942.)

103. R. A. Beaumont: *Projections of the prime-power abelian group of order p^m and type $(m-1, 1)$.*

A group H is the projection of a group G if there is a $(1-1)$ correspondence between the set of subgroups of G and the set of subgroups of H which preserves the partial ordering of the subgroups. Since R. Baer has given necessary and sufficient conditions that a group H be a projection of a group G which is the direct product of cyclic groups of order p, in the study of the projections of the prime-power abelian group G of order p^m and type $(m-1, 1)$, we may take $m>2$. It is shown that if $p>2$, the only group H, essentially different from G, which is a projection of G is the non-abelian group of order p^m containing an element of order p^{m-1}. If $p=2$ and $m>3$, the only group H, essentially different from G, which is a projection of G is the non-abelian group $\{U_1, U_2\}$ where U_1 and U_2 are subject to the sole defining relations: $U_1^{2m-1}=U_2^2=1$, $U_2U_1U_2=U^{1+2^{m-1}}$. If $p=2$ and $m=3$, a group H is a projection of G if and only if H is isomorphic to G. (Received January 16, 1942.)

104. L. L. Dines: *On the mapping of n quadratic forms.*

A transformation $x_i=Q_i(z)$, $(i=1, 2, \cdots, n)$, in which each $Q_i(z)$ is a real quadratic form in the real variables $z^1, z^2, \cdots, z^m$, maps the m-dimensional z-space into a set of points $\mathfrak{M}$ in the n-dimensional x-space. The present paper considers properties of this map $\mathfrak{M}$, and on the basis of these properties determines necessary and sufficient conditions for the existence of linear combinations $\sum \lambda_i Q_i(z)$ which are positive definite, and also conditions for the existence of such combinations which are semi-definite. The conditions are, for general n, naturally not so simple as those obtained for the case $n=2$ (this Bulletin, vol. 47 (1941), pp. 494–498). But they appear to be simpler as well as more comprehensive than those obtained by Finsler (Commentarii Mathematici Helvetici, vol. 9 (1937), pp. 188–192), and by Hestenes and McShane (Transactions of this Society, vol. 47 (1940), pp. 501–512) with which they may be compared. The paper will appear in this Bulletin. (Received December 17, 1941.)

105. H. B. Mann: *Proof of the conjecture on the density of sums of sets of positive integers.*

Let $A(n)$ denote the number of positive integers less than or equal to n in the set A. If $A+B=C$ then it is proved in this paper that: $C(n)/n \geqq \min (A(x)/x+B(x)/x)$ for $x \leqq n$. The conjecture $\gamma \geqq \alpha_1+\alpha_2+\cdots+\alpha_n$ is an immediate consequence of this result. Let $n_1, n_2, \cdots, n_r, \cdots$ be the gaps in C. Let B_r^+ be the numbers of $\bar{B}$ that are not of the form n_r-a with a in A or equal to 0. The theorem is equivalent to the following statement: If for any r, $rn_i > in_r$ and $rn_i > (B_i^+(n_i)+1)n_r$ for $i=1, 2, \cdots, r-1$, then $B_r^+(n_r) \geqq r-1$. Let $n_r - n_i = d_i$ then there always exist values i, j for which $n_r - d_i = a+b$ (a in A or equal to 0, b in B or equal to 0). Among all the b's found in this way the smallest is chosen and denoted by e_1. Let B^1 contain all numbers e_1+d_s and N^1 all numbers n_t that satisfy an equation $a+e_1+d_s=n_t$. The numbers of B^1 are added to the numbers in B and the process repeated. Thus sets $B^1, B^2, \cdots, B^i$ are formed and sets $N^1, N^2, \cdots, N^i$ and it is shown that this construction can be continued until all values n_t have been absorbed into the sets N^i. The sets B^i then contain $r-1$ different numbers of B_r^+. (Received January 15, 1942.)

106. A. R. Schweitzer: *Note on functions which generate an abstract field.*

In a previous report (this Bulletin, vol. 27 (1921), p. 249) the author stated that the following functions each generate, under suitable postulational assumptions, an abstract field: $(x+y)y$, $(1+x)y$, $(x-y)y$, $(1-x)y$. The former two functions were obtained as special instances of the function $(x+y)z$ which, it was stated, also generates a field. In this note a simple a priori proof is given by exhibiting chains of definitions of functions leading to the functions $x+y$, xy, in each instance. These chains are essentially as follows: I. $(x+y)y$: $0, 1, x+1, -x, -x+1, x-1, -(x+y), x+y, xy$. II. $(1+x)y$: $0, -1, -(1+x), -xy, -x, xy, x/y, x+y$. III. $(x-y)y$: $0, -1, -(1+x), -x, x-1, -x+1, x+1, x+y, xy$. IV. $(1-x)y$: $0, 1, 1-x, xy, x/y, x-y, x+y$. Reference is made to a report by the author, this Bulletin, vol. 26 (1920), p. 441. (Received December 29, 1941.)

107. M. F. Smiley: *Elementary similarity transformations and the rational canonical form of a matrix.*

If A is a square matrix with elements in a field F and E is an F-elementary transformation matrix, then $B=EAE^{-1}$ is said to be obtained from A by an elementary similarity transformation. A process is described involving a finite number of such transformations which replaces A by the matrix $\operatorname{diag}\{B^{(1)}, \cdots, B^{(t)}\}$ where the $B^{(i)}$ $(i=1, \cdots, t)$ are matrices in rational canonical form and the characteristic matrix of each $B^{(i)}$ has just one non-trivial invariant factor. The reduction of A to rational canonical form is then easily obtained. Examples are given to support the contention that this process simplifies the computation of the rational canonical form of A. (Received December 15, 1941.)

ANALYSIS

108. G. E. Forsythe and A. C. Schaeffer: *A remark on Toeplitz matrices.*

A doubly infinite matrix (a_{mn}) is said to be regular if for every sequence $\{x_n\}$ with limit x' the corresponding sums $y_m = \sum_n a_{mn} x_n$ are defined for all m and have the limit x'. An apparently more general definition of regularity is that the sums defining y_m exist for all sufficiently large m, depending on $\{x_n\}$, and have the limit x'. Tamarkin (this Bulletin, vol. 41 (1935), pp. 241–243) has obtained necessary and sufficient conditions for the second type of regularity. This result is obtained by elementary methods and related topics are discussed. (Received January 23, 1942.)

109. H. L. Garabedian: *Hausdorff integral transformations.*

This paper involves a study of the integral transformation $v(x) = \int_0^x u(y) d\phi(y/x)$, defining a method of summation $(H, \phi(x))$, where $u(x)$ is bounded and continuous, $x \geqq 0$, and where $\phi(x)$ is either a Hausdorff mass function or satisfies the conditions: (i) $\phi(x)$ is of bounded variation on the interval $0 \leqq x \leqq 1$, (ii) $\phi(x)$ is continuous on the interval (0, 1) except possibly at $x=1$, (iii) $\phi(0)=0$, (iv) $\phi(1)=1$. It is proved that the transformation is regular when and only when $\phi(x)$ is a Hausdorff mass function, and sufficient conditions involving the Silverman-Schmidt integral equations are obtained in order that $(H, \phi_1(x)) \supset (H, \phi_2(x))$, in the case that $\phi_1(x)$ and $\phi_2(x)$ satisfy the conditions stated above. These results are extensions of those obtained by Silverman (Transactions of this Society, vol. 26 (1924), pp. 101–112). (Received January 10, 1942.)

110. A. M. Gelbart: *Functions of two variables with bounded real parts in domains not equivalent to the bicylinder.*

Let $f(z_1, z_2)$ be regular in the interior of a finite four-dimensional domain $\mathfrak{M}^4$, bounded by certain analytic hypersurfaces, and in general not equivalent to the bicylinder, and let $f(z_1, z_2)$ have a bounded real part in $\mathfrak{M}^4$. These domains were first considered by Bergman, and are termed by him, domains with distinguished boundary surfaces. An upper bound for $|f(z_1, z_2)|$ is obtained in terms of only max $Re\, f(z_1, z_2)$ in $\mathfrak{M}^4$, $f(0, 0)$ and the domain considered. From a formula for $\partial^{m+n} f(z_1, z_2)/\partial z_1^m \partial z_2^n$ in $\mathfrak{M}^4$, previously obtained by the author (Transactions of this Society, vol. 49 (1941), pp. 199–210), an upper bound is also obtained for $|\partial^{m+n} f(z_1, z_2)/\partial z_1^m \partial z_2^n|$, again in terms of only max $Re\, f(z_1, z_2)$ in $\mathfrak{M}^4$, $f(0, 0)$ and the domain. These results depend upon the establishment of a form of the Schwarz lemma in $\mathfrak{M}^4$ for two variables. (Received January 29, 1942.)

111. H. J. Greenberg and H. S. Wall: *Hausdorff means included between (C, 0) and (C, 1).*

It is shown that if $\phi(u)$ is any function of bounded variation on the interval $0 \leqq u \leqq +\infty$ such that $\phi(+\infty) - \phi(0) = 1$, then the function $\alpha(z) = \int_0^\infty d\phi(u)/(1+zu)$ is a regular moment function; and that when $\phi(u)$ is further restricted to be monotone