GEOMETRY

75. L. M. Blumenthal: *Metric characterization of n-dimensional elliptic space $\mathcal{E}_{nr}$.* Preliminary report.

The objective is the characterization of the elliptic metric by means of relations between mutual distances of points in certain finite subsets of the space. If δ-supplementation denotes the process by which a semi-metric Σ^* arises from a semi-metric Σ of diameter d upon replacing arbitrary distances pq in Σ by $\delta - pq$, $\delta \geqq d$, and identifying points with zero distance in Σ^*, then $\mathcal{E}_{nr} = \sup_{\pi r} S_{nr}$, where S_{nr} is the metrically convex spherical surface of radius r and dimension n. It is proved that semi-metric Σ is congruently contained in $\mathcal{E}_{nr}$ if and only if $p, q \in \Sigma$ implies $pq \leqq \pi r/2$ and there exists a $\sup_{\pi r} \Sigma$ congruently contained in S_{nr}. A semi-metric m-tuple $p_1, p_2, \cdots, p_m$ with $p_i p_j \leqq \pi r/2$ is congruently contained in $\mathcal{E}_{nr}$ if and only if a symmetric square matrix (ϵ_{ij}), $\epsilon_{ii} = 1$, $\epsilon_{ij} = 1$ $(i, j = 1, 2, \cdots, m)$, exists such that the determinant $|\epsilon_{ij} \cos (p_i p_j / r)|$ has rank not exceeding $n+1$ with all nonvanishing principal minors positive. This puts in algebraic form the determination of congruence indices for the $\mathcal{E}_{nr}$, at least with respect to finite semi-metric sets. (Received October 24, 1941.)

76. Nathaniel Coburn: *Unitary curves in unitary space.*

The question of the existence of an arc length parameter for a unitary curve K_1 imbedded in an n-dimensional unitary space K_n is discussed. First, the familiar formula for arc length element (ds^2) is generalized. Then, it is shown that if and only if K_1 possesses a natural parameter, does an arc length parameter which is an analytic function of the curve parameter exist. In fact, ∞^1 such parameters exist; all have the same absolute value but different moduli. If the curve parameter is real (K_1 reduces to X_1), then again ∞^1 such arc length parameters exist. One and only one of these parameters is real and positive; this parameter is the one commonly associated with X_1 in K_n. The remainder of the paper is concerned with determining those K_n into which can be imbedded various classes of K_1 which possess an arc length parameter (such K_1 are denoted by U_1). The principal result is: If the metric tensor of K_n is not of rank one, then those unitary K_1 which satisfy a system of differential equations of the third or higher order in the parameter are not U_1. (Received October 24, 1941.)

77. N. A. Court: *On the theory of the tetrahedron.*

A "quasi-polar" sphere (Q) may be associated with the general tetrahedron (T) having for center the Monge point M of (T) and for the square of its radius one third of the power of M for the circumsphere (O) of (T). The following two propositions may serve as samples of the many properties of (Q): The "quasi-polar" sphere is coaxial with the circumsphere (O) and the twelve point sphere (L) of (T); The polar reciprocal tetrahedron of (T) with respect to the sphere (Q) is circumscribed about the medial tetrahedron of (T). A second sphere (G) may be related to (T) having for center the centroid G of (T) and for the square of its radius one forty-eighth of the sum of the squares of (T). The sphere (G) is orthogonal to (Q) and is coaxial with (Q), (O), and (L). The four spheres having for centers the vertices of (T) and orthogonal to (Q) cut the spheres having for diameters the respective medians of (T) along four circles lying on the same sphere, namely the sphere (G) of (T). In the special case when the tetrahedron becomes orthocentric, the spheres (Q) and (G) become, respectively, the polar sphere and the first twelve point sphere of the orthocentric tetrahedron. (Received November 21, 1941.)

78. J. W. Peters: *The euclidean geometry of the n-dimensional simplex.*

In this paper theorems associated with the triangle and the tetrahedron are extended to the n-dimensional simplex formed by $n+1$ points in a euclidean space of n dimensions. The centroid and Monge point of the simplex as well as the centroids and Monge points of the faces are defined. The following extension of Mannheim's theorem for a tetrahedron is proved. The $n+1$ planes determined by the $n+1$ altitudes of the simplex and the Monge points of the corresponding faces meet in the Monge point of the simplex. A hypersphere on the centroids of the faces is discussed. It is shown that this hypersphere passes through $3(n+1)$ points associated with the simplex and has a number of properties similar to the nine point circle associated with a triangle and with the twelve point sphere associated with a tetrahedron. (Received November 8, 1941.)

79. J. L. Vanderslice: *Invariant theory of vector pencil fields.*

At each point $(x^1, \cdots, x^n)$ of a space subject to general coordinate transformations is associated a pencil of contravariant vectors, $\xi^i(x^1, \cdots, x^n, u)$. The parameter u of the pencil is normalized to give an invariant metric parameter x^0 associated with each coordinate system x^i and transforming like the gauge variable of generalized projective geometry. The principal result is the discovery of an affine connection with components which are functions of x^α $(\alpha=0, 1, \cdots, n)$ and a method of covariant differentiation of tensor functions of x^α. A study of the equivalence problem then leads to a complete set of tensor invariants for the vector pencil field. (Received November 17, 1941.)

80. André Weil and C. B. Allendoerfer: *A general proof of the Gauss-Bonnet theorem.*

The following generalized Gauss-Bonnet theorem was recently proved independently by Allendoerfer and Fenchel: If a closed Riemann manifold R_n of even dimension can be made a subspace of an euclidean space then $\int KdO = 1/2\omega_n\chi$ where K is the total curvature of the manifold, ω_n is the area of an n-dimensional sphere, and the integration is over the manifold. The present paper removes the restriction that R_n be a subspace of an euclidean space. To do this R_n is subdivided into simplices each of which is small enough to have an isometric euclidean imbedding. The method of tubes is applied to these subdivisions separately, special attention being paid to their boundaries. The terms resulting from the boundaries are found to be intrinsic and drop out when the simplices are reassembled to form R_n. (Received November 26, 1941.)

Logic and Foundations

81. E. C. Berkeley: *Application of symbolic logic to punch card operations.*

This paper discusses the analysis of operations with punched tabulating cards, and also to some extent operations with handwritten cards, as taking place in a large life insurance company, for purposes of valuing policies, computing, recording, and summarizing payments, and so on. The chief instrument of analysis is a system of coding, constructed using symbolic logic and other techniques. The system of coding is exhibited in part, and examples of the coding are given. Some related problems of