477. H. S. Wall: *The behavior of certain Stieltjes continued fractions near the singular line.*

The author shows that the function $f(z)$ represented by a continued fraction of the form $1/1+g_1z/1+(1-g_1)g_2z/1+(1-g_2)g_3z/1+\cdots$ in which $0<g_n<1$, $n=1, 2, 3, \cdots$, and the series $\sum|1-2g_n|$ converges, approaches a finite limit $\alpha(s)$ as $z\rightarrow -s$, $s>1$, along any path in the upper half-plane, and the limit $\bar{\alpha}(s)$, the complex conjugate of $\alpha(s)$, as $z\rightarrow -s$ from the lower half-plane. The function $\alpha(s)$ is continuous and not real, $s>1$. (Received August 18, 1941.)

Applied Mathematics

478. Arthur Korn: *On the problem of pulsating spheres in a liquid.*

The problem of the pulsating spheres treated for the first time almost 80 years ago by the Norwegian mathematician C. A. Bjerknes and chosen by the author as the point of departure for a mechanical theory of gravitation and the electromagnetic field, must be considered as a fundamental problem of mathematical physics. Interactions inversely proportional to the square of the distance can solely be produced by such spheres changing their volumes periodically with approximately the same frequency in an incompressible or approximately incompressible fluid. The sign of the forces of interaction (attraction in the case of equal phases, repulsion in the case of opposite phases) is a certain difficulty for the hypothesis that electrical particles are pulsating spheres. The sign can be changed, if one imposes on the spheres the condition that they maintain the amplitudes of their pulsations at any displacement of the spheres, a condition which is not imposed in Bjerknes' problem. The calculations are a little complicated, if one has to find out the pressure at the surfaces of the spheres and if one derives from these pressures the forces in question. Here is given a rather simple way of deriving the forces from the energy of the liquid in the case of displacements of the spheres, and one can easily understand the change of the sign, if one imposes on the spheres the condition that they maintain the amplitudes of their pulsations at any displacement. (Received September 22, 1941.)

Geometry

479. George Comenetz: *Isotropic curves on a surface.*

Let S be a regular analytic surface in complex three-dimensional euclidean space, and O a point of S at which the tangent plane T is isotropic. Such a point is singular for the differential equation $ds^2=0$ of the isotropic net of curves on S. The problem is to describe the regular analytic isotropic curves on S that pass through O. Let L be the isotropic straight line in T through O, and p its order of contact with S. Draw an arbitrary curve F on S having contact of order p with L at O. Draw the normal to S at a point V of F, and let q be the "order of coincidence" of the normal with its limit L, as V approaches O. Then if $q>p/2$ the number of isotropic curves is 0 or 2, if $q=p/2$ it is 1 or 2 or $1+\infty^1$, and if $q<p/2$ it is 2. The conditions under which the different possibilities are realized, and the orders of contact of the curves with L and one another, are stated. (Received September 15, 1941.)

480. J. J. DeCicco: *Bi-isothermal systems of curves.*

A generalization of the concept of isothermal family of curves in four-dimensional space S_4 is obtained in this paper. Kasner has termed the correspondences of S_4, defined by pairs of functions of two complex variables, the pseudo-conformal group G. This is characterized by the preservation of Kasner's pseudo-angle between a curve and a hypersurface at their common point. Any system of ∞^3 curves which is pseudo-conformally equivalent to a parallel set of lines is said to be bi-isothermal. Any such system consists of ∞^2 isothermal families, each family living on a conformal surface. A characteristic property of bi-isothermal systems is that the pseudo-angle between this system and any parallel pencil of hyperplanes is a biharmonic function. Finally G is characterized within the group of point transformations by the preservation of the totality of all bi-isothermal systems. (Received September 29, 1941.)

481. Walter Prenowitz: *Descriptive geometries as multigroups.*

Let G be the set of points of a descriptive space of arbitrary dimension. In G, define $a+b$ as the set of points between a and b if $a\neq b$, and $a+a$ as a. Then G becomes an abelian multigroup of special type. Convex sets and linear manifolds appear respectively as semigroups (subsets closed under $+$) and subgroups of G. Half-spaces (rays, half-planes, and so on) are cosets, when the latter are properly defined, and some of the elementary properties of half-spaces follow from a coset decomposition theorem. Elementary properties of these three types of figures are derived algebraically and many familiar group theoretic concepts as factor group, homomorphism, congruence relation are used in the development. (Received September 29, 1941.)

482. C. V. Robinson: *Spherical theorems of Helly type and congruence indices of spherical caps.*

The theorems mentioned in an earlier abstract (47-1-67) have been extended to the n-sphere. The principal theorem of Helly type reads: If each $n+k+2$ members of a family of convex subsets of the n-sphere intersect and if one member contains no k-dimensional great hypersphere then there is a point common to all the sets of the family. A study is also made of the congruence indices of spherical caps of various radii with respect to the class of semi-metric spaces. For example, it is found that a cap of spherical radius $\rho<\pi r/2$ of the 2-sphere of radius r will contain isometrically any semi-metric space of more than 6 points if every quadruple of the space is isometrically imbeddable in the cap, that is, the cap has the congruence indices [4, 2]. (Received September 23, 1941.)

Statistics and Probability

483. J. F. Daly: *A problem in estimation.*

Consider a normal population in which each individual is characterized by the variates $y_1, \cdots, y_p, y_{p+1}, y_{p+2}$. Suppose that the latter two are not directly observable, but that for given values of y_{p+1}, y_{p+2} the first set of y's is independently distributed about the "regression line" $y_k=y_{p+1}+ky_{p+2}$ $(k=1, \cdots, p)$ with a common variance σ^2. For each individual, one can thus determine values $\hat{y}_{p+1}, \hat{y}_{p+2}$ from the observed $y_1, \cdots, y_p$, using the method of least squares. Assuming a similar relation between the expected values of $y_1, \cdots, y_{p+2}$ in the original population, these estimates $\hat{y}_{p+1}, \hat{y}_{p+2}$ are of course unbiased. However, if we calculate these $\hat{y}$'s for each individual of a sample of N and substitute them in the Pearson product-moment correlation formula the estimate of the correlation between y_{p+1} and y_{p+2} thus obtained is somewhat biased. The bias depends on the number of observable y's and on the size of the variances and covariances of y_{p+1}, y_{p+2} relative to σ^2. (Received September 2, 1941.)