than one. The implications of the non-existence of conjugate points are also investigated in the case of surfaces of the topological type of the torus. It is found that to a large extent the geodesics have the properties of those on a manifold for which the Gaussian curvature vanishes identically. If an added hypothesis concerning the non-existence of focal points is invoked, it can be shown that the curvature does vanish identically. (Received June 7, 1941.)

426. Abraham Schwartz: *A consequence of the Ricci equations for Riemannian manifolds.*

The paper considers the normal spaces at a point of a riemannian manifold of m dimensions which is imbedded in a manifold of constant curvature of n dimensions, $V_m \subset S_n$. With each normal space, $x=1, \cdots, k$, there is associated a curvature tensor $H(x)$ in a well known way, each tensor $H(x)$ being related to the preceding one, $H(x-1)$, by a Ricci equation. From these Ricci relationships information concerning the dimensionalities of the various normal spaces can be deduced. For instance, if the first normal space is one-dimensional, then the second normal space is zero or one-dimensional according as the rank of the second fundamental form is $\geqq 2$ or <2; if the first normal space is two-dimensional, then the second normal space is zero-, one-, or two-dimensional depending on the nature of the elementary divisors of the two second fundamental forms. Analogous theorems are true for the other normal spaces. (Received July 24, 1941.)

427. S. M. Ulam and D. H. Hyers: *On approximate isometries.* Preliminary report.

Let E and F be metric spaces and let ϵ be a positive number. The question studied is the following. If $T(x)$ is a continuous transformation of E into F such that $|\rho(T(x), T(y))-\rho(x, y)| \leqq \epsilon$ for all x and y, then does there exist an isometric transformation $S(x)$ of E into F such that for all, x, $\rho(S(x), T(x)) \leqq k\epsilon$, where k is a positive constant, depending only on the spaces E and F? The question is answered in the affirmative for the cases in which E and F are Hilbert spaces or finitely dimensional euclidean spaces. (Received July 30, 1941.)

Logic and Foundations

428. Garrett Birkhoff: *Metric foundations of geometry.*

New derivations of fundamental theorems of euclidean, hyperbolic and spherical geometry, with particular reference to the lattice of subspaces, are given. The postulates are: (1) metric postulates of Fréchet, (2) straight line postulates of Menger, in a weakened form valid in any riemannian geometry, (3) the postulate of Pasch. Thus the local compactness postulate of Busemann's system is dispensed with, and his straight line postulates are weakened; on the other hand, the postulate of Pasch is not restricted to 3 points. The proofs are elementary throughout. (Received July 7, 1941.)

429. Henry Blumberg: *A reconsideration of the paradoxes in the theory of sets.*

Proceeding from an intuitive bias that it should be possible to eliminate the paradoxes in question along the line of natural human understanding, without reforming our familiar logic or invoking ingenious technical or professional stratagems, the author, upon carefully re-examining these paradoxes, has reached the following conclusions: (a) None of them arises if the would-be paradox maker is estopped from employing reasoning that may in fairness be rejected by people of sound understanding. (b) A conception of set may be delineated which accords with natural expectations and by means of which we may build a reliable theory of sets comprehending virtually all the results that have been subjected to attack. The relation is indicated which this conception bears to the basic conceptions, in the matter at issue, of the intuitionists (Brouwer and his school), the formalists (Hilbert and his school), the logicians (Russell and others), and the postulationists (Zermelo, Fraenkel). (Received July 29, 1941.)

430. A. R. Schweitzer: *Concerning general abstract relational spaces. III.*

On the basis of the general abstract relational space $S_{n+1}(G, H)$ $(n=1, 2, 3, \cdots)$, $G=H=$symmetric group on $n+1$ variables and certain axioms elaborating this space, the author constructs axioms for the (finite) algebra of logic analogous to his system ${}^{n+1}K_{n+1}$ for the foundations of geometry. For $n=3$ the elements of S_4 are α, β, γ, δ, λ, μ, ν, ω with $\alpha\beta\gamma\delta\ K$, that is, $\alpha\beta\gamma\delta\ K\ \alpha\beta\gamma\delta$. Axioms elaborating S_4 are the following: 1. $\alpha\beta\gamma\delta\ K \supset \lambda\mu\nu\omega\ K$. 2. $\alpha\beta\gamma\delta\ K$ and $\xi \supset \xi\beta\gamma\delta\ K$ or $\alpha\xi\gamma\delta\ K$ or $\alpha\beta\xi\delta\ K$ or $\alpha\beta\gamma\xi\ K$. 3. $\alpha\beta\gamma\delta\ K$ and $\lambda\mu\nu\omega\ K$ and ξ, $\eta \supset \alpha\lambda\xi\eta$ not K, $\beta\mu\xi\eta$ not K, $\gamma\nu\xi\eta$ not K, $\delta\omega\xi\eta$ not K. The complete set of 2^4 K tetrads is expressed as a reflexive formal sum $\sum(I)$ and classified into subsums: $\sum(\xi)$ is the sum of all K tetrads containing ξ, and so on. If $\xi\eta\zeta\tau\ K$, then $\sum(\xi\eta\zeta\tau)=\xi\eta\zeta\tau$. The existence of a unique "empty" sum $\sum(0)=\sum(\alpha\lambda)=\sum(\beta\mu)=\sum(\gamma\nu)=\sum(\delta\omega)$ is assumed. The summands of the various $\sum$'s are replaced by their corresponding expressions in terms of $\sum$ and the $\sum$'s are then represented as products $\sum(\xi\eta)=\sum(\xi)\times\sum(\eta)$, and so on. The preceding continues a paper reported in this Bulletin (abstract 46-9-438). (Received July 22, 1941.)

STATISTICS AND PROBABILITY

431. K. J. Arnold: *On spherical probability distributions.*

Two methods of correspondence for circular distributions to the normal error function have led to non-constant absolutely continuous functions (see F. Zernike, *Handbuch der Physik*, vol. 3, pp. 477–478). The corresponding distributions for the sphere are found. The case of diametrical symmetry for both circle and sphere is discussed. Tables of the probability integrals involved are given and an application in geology is included. (Received July 31, 1941.)

432. I. W. Burr: *Cumulative frequency functions.*

Frequency and probability functions play a fundamental role in statistical theory and practice. They are, however, often inconvenient and difficult to use, since it is necessary to integrate or sum to find the probability for a given range. Theoretically the cumulative or integral frequency function would seem to be better adapted to determining such probabilities, since the latter can be found simply by a subtraction. The aim of this paper is to make a contribution toward the direct use of cumulative frequency functions. Some general properties and theory of cumulative functions are presented with particular emphasis upon certain moment functions adapted to such direct use. Both continuous and discrete cases are included. A list of possible cumulative functions is given and a particular one, $F(x)=1-(1+x^c)^{-k-1}$, discussed fully. This function has properties which make it practicable and adaptable to a wide variety