ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

353. Stefan Bergman: *On a generalized Green's function and certain of its applications.*

A theorem of Blaschke states: $\sum_{\nu=1}^{\infty} \lg |a_\nu| > -\infty$, $|a_\nu| < 1$, is the necessary and sufficient condition for the existence of a nonnegative harmonic function $H(z)$, $H(0) < \infty$, $z \varepsilon \mathfrak{C}^2 - \mathfrak{S}_\nu \{a_\nu\}$, $\mathfrak{C}^2 = \mathrm{E}[|z| < 1]$ which possesses the property that $H(z) + \log|z - a_\nu|$, $\nu = 1, 2, \cdots$, is regular at $z = a_\nu$. Let $\mathfrak{M}^4 = \mathrm{E}[|z_2| < 1, z_1 \varepsilon \mathfrak{B}^2(z_2)]$, $\mathfrak{B}^2(z_2) = \mathrm{E}[z_1 = sh(z_2, \lambda), 0 \leqq s < 1, 0 \leqq \lambda \leqq 2\pi]$, $h(z_2, \lambda)$, $h(z_2, 0) = h(z_2, 2\pi)$, being for every λ an analytic function of z_2, $z_2 \varepsilon \overline{\mathfrak{C}}^2$. In the paper (Compositio Mathematica, vol. 6 (1939), pp. 307–335) there was introduced for every domain $\mathfrak{M}^4$ an "extended class of functions" possessing properties analogous to those of harmonic functions. If $g(z_1, z_2)$ is in $\mathfrak{M}^4$ an analytic function, it is then possible to construct the "Green's function of the extended class" $\Gamma(z_1, z_2; g; \mathfrak{M}^4)$ vanishing on the boundary surface $\mathrm{E}[|z_2| = 1, z_1 = h(z_2, \lambda), 0 \leqq \lambda \leqq 2\pi]$ such that $\Gamma(z_1, z_2; g; \mathfrak{M}^4) + \lg|g(z_1, z_2)|$ is a function of extended class. Let $g_\nu(z_1, z_2)$ designate a set of functions analytic in $\mathfrak{M}^4$ satisfying a certain additional restriction concerning the distribution of zero surfaces. The condition $\sum_{\nu=1}^{\infty} \Gamma(0, 0; g_\nu; \mathfrak{M}^4) < \infty$ is necessary and sufficient for the existence of a nonnegative function $H(z_1, z_2)$, $H(z_1, z_2) < \infty$ for $(z_1, z_2) \varepsilon \mathfrak{M}^4 - \mathfrak{S}_\nu \mathfrak{G}_\nu^2$, $\mathfrak{G}_\nu^2 = \mathrm{E}[g_\nu(z_1, z_2) = 0, (z_1, z_2) \varepsilon \mathfrak{M}^4]$ such that for every n, $H(z_1, z_2) + \sum_{\nu=1}^{n} \lg |g_\nu(z_1, z_2)|$, belongs to the extended class of functions of $\mathfrak{M}_{n0}^4$, $n_0 \leqq n$, where $\lim_{n \to \infty} \mathfrak{M}_n^4 = \mathfrak{M}^4$. (Received March 28, 1940.)

354. L. M. Blumenthal: *A new concept in distance geometry, with applications to spherical subsets.*

If $\sigma \subset S$, a semimetric space, then S has σ-relative congruence indices $\{n, k\}$ with respect to a given class (Σ) of semimetric spaces provided any space Σ of (Σ) with more than $n+k$ pairwise distinct points is congruently contained in S whenever each n of its points is congruently contained in σ. This concept gives rise to new problems in distance geometry. For $\sigma = S$ the indices $\{n, k\}$ are called merely congruence indices of S with respect to (Σ). The notions of congruence and quasi congruence orders introduced by Menger correspond to congruence indices $\{n, 0\}$, $\{n, 1\}$, respectively, while $\{n, 2\}$ signifies a property weaker than quasi congruence order n but stronger than quasi congruence order $n+1$. Let $\kappa_{2,\rho}$ denote a cap with spherical radius ρ of the sphere $S_{2,r}$. It is shown that (1) if $\sigma = \kappa_{2,\pi r/2}$, with base circle removed, then $S_{2,r}$ has σ-relative congruence indices $\{4, 2\}$ with respect to all semimetric spaces (Σ); (2) the closed cap $\kappa_{2,\rho}$, $\rho < \pi r/2$, has congruence indices $\{4, 2\}$ with respect to (Σ); (3) $\kappa_{2,\rho}$, $\rho < r \cos^{-1} 1/3$, has indices $\{4, 1\}$. Analogous theorems for caps of $S_{n,r}$ are obtained. (Received April 12, 1940.)

355. S. S. Cairns: *Triangulated manifolds which are not Brouwer manifolds.*

This paper relates to problems involving Brouwer manifolds, differentiable manifolds, and polyhedral representations of complexes. A *Brouwer m-manifold* (L. E. J. Brouwer, Mathematische Annalen, vol. 71 (1912), pp. 97–115) is a triangulated topological m-manifold such that each star has a piecewise linear homeomorphic image in an euclidean m-space E^m. (I) A triangulated topological m-manifold is not necessarily a Brouwer manifold if $m>3$. This is one consequence of a constructed triangulation of an m-simplex ($m=3, 4, 5, \cdots$) not homeomorphic to any rectilinear triangulation of it. Other consequences follow. (II) Triangulated m-spheres exist ($m=3, 4, 5, \cdots$) which cannot be represented as convex polyhedral spheres in an E^{m+1}. (III) The lowest-dimensional euclidean space in which a complex K has a nonsingular polyhedral representation is not, for every K, invariant under subdivision. (IV) For every $m>3$, there exist stars of simplexes which are m-cells but which do not admit transversal $(n-m)$-planes, no matter how they are polyhedrally represented in any E^n. A plane is *transversal* to a point set if it makes angles bounded away from zero with secants of the set. (V) Brouwer's definition of an m-manifold (loc. cit.) is not invariant ($m>3$) under subdivision. For the connection with questions of differentiability, see abstracts 45-3-110 and 45-9-306, the results of which are to appear in the Annals of Mathematics. (Received March 29, 1940.)

356. J. L. Coolidge: *Analytic systems of central conics in space.*

Systems of central conics in three-space have been little studied. Spottiswoode and Johnson have studied algebraic systems and there is a paper by Blutel dealing with surfaces generated by conics of a particular type. In the present paper there is a study of general one- and two-parameter analytic systems. The technique is that of the moving tetrahedron first developed by Darboux and employed in more recent times by Cartan. (Received April 9, 1940.)

357. Richard Courant: *On a generalized form of Plateau's problem.*

From a geometrical viewpoint it is natural to replace the Plateau problem, in which a continuous and monotonic description of the prescribed boundary is required, by another problem where no condition of this type is stipulated. For the solution of this generalized problem, methods quite different from those usually used for Plateau's problem are needed. The main objective of the paper is to show that the solution of the ordinary Plateau problem also solves the new problem, while the converse is not necessarily true. (Received April 11, 1940.)

358. A. González Domínguez: *Some theorems on the Hermite singular kernel.*

Let $g(x)$ be a function of bounded variation in $(-\infty, \infty)$ normalized in the usual way, and let $K(x, y, r)$ be the singular kernel which appears in the Abel summation of Hermite series. Then the following equality holds: $\lim_{r\to 1} \int_0^a dy \int_{-\infty}^{\infty} K(x, y, r)dg(x) = g(a)-g(0)$. This result, and others of a similar character, are used to obtain a uniqueness theorem for Hermite-Stieltjes series, a new limit-theorem for distributions functions, and also to solve the problem of the "classes" for Hermite series. (Received April 4, 1940.)

359. J. E. Eaton: *On the extension of groups. I. Normal extensions. II. Non-normal extensions.*

I. The problem of finding all groups $\mathfrak{G}$ such that $\mathfrak{G}/\mathfrak{N}\cong\mathfrak{H}$ for given groups $\mathfrak{H}$ and $\mathfrak{N}$ has been considered by Schreier. In this paper Schreier's n^3 associativity conditions on n^2 elements of $\mathfrak{N}$ are replaced by kn more complicated conditions on rn elements, wherein n is the number of elements of $\mathfrak{H}$, k the number of generating relations defining $\mathfrak{H}$, and r the number of generators. The problem of obtaining all non-redundant solutions of these conditions is then considered. II. By using the results of a previous paper (abstract 45-9-310) one may formulate the problem of non-normal extensions of groups: *to determine all groups* $\mathfrak{G}$ *such that* $\mathfrak{G}/\mathfrak{H}\cong\mathfrak{C}$ *where* $\mathfrak{H}$ *is a given group and* $\mathfrak{C}$ *a given "cogroup."* If one calls those groups $\mathfrak{G}$ for which no proper subgroup of $\mathfrak{H}$ is normal in $\mathfrak{G}$ a *purely non-normal extension*, the problem of non-normal extensions may be reduced to purely non-normal ones followed by normal extensions. Conditions for purely non-normal extensions, analogous to those of Schreier for the normal case, are derived. (Received April 13, 1940.)

360. D. W. Hall: *On arc and tree preserving transformations.*

It is shown that every single valued transformation $T(A)=B$, where A and B are cyclic locally connected continua, which is arc preserving and tree preserving is necessarily a homeomorphism. No assumptions are made as to the continuity of the transformation T. (Received April 26, 1940.)

361. E. R. Lorch: *The integral representation of weakly almost-periodic transformations in reflexive vector spaces.*

As announced previously (abstract 45-11-401), a weakly almost-periodic transformation V in a reflexive space $\mathfrak{B}$ is characterized by the relation $\|V^nf\|\leqq K\|f\|$, $n=0, \pm 1, \pm 2, \cdots$. For such a w.a.p. V, an integral representation is given which is the exact counterpart of the known representation $U=\int e^{i\lambda}dE(\lambda)$ for a unitary transformation U in a Hilbert space. Specifically it is proved: I. There exists a family of pairs of closed linear manifolds $\{\mathfrak{C}_\lambda, \mathfrak{F}_\lambda\}$, $-\infty<\lambda<\infty$, with the following properties: (a) $\mathfrak{C}_\lambda$ and $\mathfrak{F}_\lambda$ have only 0 in common; together they span $\mathfrak{B}$. (b) The pair $\{\mathfrak{C}_\lambda, \mathfrak{F}_\lambda\}$ reduces V. (c) $\mathfrak{C}_\lambda\supset\mathfrak{C}_\mu$ for $\lambda>\mu$; $\mathfrak{F}_\lambda\subset\mathfrak{F}_\mu$ for $\lambda>\mu$. (d) $\mathfrak{C}_{-\pi}=0$, $\mathfrak{F}_{-\pi}=\mathfrak{B}$, $\mathfrak{C}_\pi=\mathfrak{B}$, $\mathfrak{F}_\pi=0$. II. Given an $\epsilon>0$, there exists a set of numbers $\lambda_1, \lambda_2, \cdots, \lambda_n$ and closed linear manifolds $\mathfrak{A}_{\lambda_j}$, $j=1, \cdots, n$, such that (a) $\mathfrak{A}_{\lambda_j}$ lies in $\mathfrak{C}_{\lambda_j+\epsilon}$ and in $\mathfrak{F}_{\lambda_j-\epsilon}$, (b) on $\mathfrak{A}_{\lambda_j}$, $\|(V-e^{i\lambda})f\|\leqq\epsilon\|f\|$, (c) $\mathfrak{B}$ is spanned by the $\mathfrak{A}_{\lambda_j}$, $j=1, \cdots, n$. III. The spectrum of V is competely characterized by the behavior of $\{\mathfrak{C}_\lambda, \mathfrak{F}_\lambda\}$. That is, for $-\pi<\lambda<\pi$, $e^{i\lambda}$ is in the point spectrum, the continuous spectrum, or the resolvent set of V if and only if λ is a point of discontinuity, of continuity, or of constancy, respectively, of $\{\mathfrak{C}_\lambda, \mathfrak{F}_\lambda\}$. The integral representation and spectral properties of the transformation $H=-i(V-I)(V+I)^{-1}$ are obtained. (Received April 25, 1940.)

362. A. N. Lowan and Jack Laderman: *On the distribution of errors in the computation of tables by differences.*

The values of a function may often be computed conveniently from the leading differences up to the order $r-1$, and the rth differences (which are not assumed to be constant) for all the required arguments. The values obtained by building up the table from the above differences are equal to those obtained from the formula $u_n=u_0+C_{n,1}\Delta u_0+C_{n,2}\Delta^2u_0+\cdots+C_{n,r-1}\Delta^{r-1}u_0+\sum_{i=0}^{n-r}C_{n-i-1,r-1}\Delta^ru_i$. Expressions are given for the maximum possible error in u_n, for the distribution of the error, and for the variance of the distribution when the differences have been rounded in the kth decimal place. The above results were obtained in the course of work done by the Project for the Computation of Mathematical Tables, Work Projects Administration, New York City. (Received April 10, 1940.)

363. A. N. Lowan and Hyman Serbin: *On upper bounds for the remainder in the evaluation of differences.*

The nth difference of a function $f(x)$ defined over the interval from x_0 to x_0+nh may be expressed in the elegant form $\Delta^n f(x)=\int_0^h\int_0^h \cdots \int_0^h f^{(n)}(x+\xi_1+\xi_2+\cdots+\xi_n)d\xi_1 d\xi_2\cdots d\xi_n = h^n\int_0^1\int_0^1\cdots\int_0^1 f^{(n)}(x+\xi_1+\xi_2+\cdots+\xi_n)d\xi_1 d\xi_2\cdots d\xi_n$. This leads to $\Delta^n f(x)=h^n f^{(n)}(x)+\sum_{h=1}^{p}(k!)^{-1}h^{n+k}f^{(n+k)}(x)\int_0^1\cdots\int_0^1(\xi_1+\cdots+\xi_n)^k d\xi_1\cdots d\xi_n + [h^{n+p+1}/(p+1)!]\int_0^1\cdots\int_0^1(\xi_1+\cdots+\xi_n)^{p+1}f^{(n+p+1)}(x+\theta(\xi_1+\cdots+\xi_n))d\xi_1\cdots d\xi_n$. If the expression for $\Delta^n f(x)$ is truncated after the term involving the derivative of order $n+p$, an upper bound for the truncating error is given by $[h^{n+p+1}/(p+1)!]\{f^{(n+p+1)}\}\int_0^1\cdots\int_0^1(\xi_1+\cdots+\xi_n)^{p+1}d\xi_1\cdots d\xi_n$ where $\{f^{(n+p+1)}\}$ is an upper bound of $f^{(n+p+1)}(x)$ in the given interval. An upper bound of the last integral is given by n^{p+1}. The application of this formula to x^{n+m} (where m is a positive integer) leads to an expression of Milne-Thomson's Bernoulli numbers $B^{(-n)}$ in terms of the above repeated integrals. The above results were obtained in the course of work done by the Project for the Computation of Mathematical Tables, Work Projects Administration, New York City. (Received April 6, 1940.)

364. M. H. Martin: *Quasi-Lagrangian systems.* Preliminary report.

Non-conservative dynamical systems $L_{\dot{q}_r}-L_{q_r}+k(t)L_{\dot{q}_r}=0$ are studied. Such systems have properties allied to Lagrangian systems ($k\equiv 0$) and are termed *quasi-Lagrangian* systems. The restricted problem of three bodies involving the motion of an infinitesimal mass attracted by two finite masses moving in accordance with a preassigned solution of the two body problem leads to a quasi-Lagrangian system, as do certain dynamical systems containing frictional forces. The equations of motion may be put in the variational form $\delta\int_{t_0}^{t_1}e^l L dt=0$, ($\dot{l}=k$) and in the canonical form $g_r=H_{p_r}$, $\dot{p}_r=-H_{q_r}-kp_r$ with the partial differential equation $S_t+kS+H(t, q_r, S_{q_s})=0$ playing the role of the Hamilton-Jacobi partial differential equation. The flow in the phase space is not incompressible, the volume V of a region being given in terms of its volume V_0 at $t=t_0$ by the relation $V=e^{-n(l-l_0)}V_0$. When $L_t\equiv 0$ and k is constant there exist generalizations of the energy integral and of the principle of Mapertuis. At least half the characteristic exponents of an equilibrium solution have negative (positive) real parts if $k>0$ ($k<0$). If L contains no linear terms in $\ _r$, there are no periodic solutions and positively (negatively) stable motions tend uniformly towards equilibrium points.The nature of flow in the neighborhood of equilibrium points is investigated. (Received April 12, 1940.)

365. H. J. Miser: *Regions and their "patterns" in conformal mapping.*

Let the circle $|z|<1$ be mapped by $w=f(z)$, ($f(0)=0$), on a plane region S, the map of $|z|<r<1$ being S_r. Let $w_0=T(w)$ be analytic in S, and let $T(0)=0$. If w_0 is in S for all w in S, then S is said to have the property T. It has been proved (L. R. Ford, Duke Mathematical Journal, vol. 1 (1935), pp. 103–104) that if S has the property T, so also does S_r. This paper considers the function $w_0=tw$, and studies the set of values Σ in the t-plane, which is called the "pattern" of S, for which S has the property T. The set Σ contains 0 and 1, lies in $|t| \leqq 1$, is closed, and is contained in the pattern Σ_r of S_r. (Received April 5, 1940.)

366. I. E. Segal: *Certain topological rings.*

The rings considered are Banach spaces which are rings, with the norm having the property $|ab| \leqq |a| \cdot |b|$, for example, the ring of all Fourier-Stieltjes transforms. Theorems are derived which connect the properties of an element modulo the closed maximal ideals of the ring with the existence of an inverse to the element, or the existence of any given analytic function of the element. Some special cases are treated. (Received March 29, 1940.)

367. E. W. Titt: *On the characteristic theory of a first order partial differential equation.*

This paper can be regarded as a continuation of a previous study of the relation between the $(n-1)$-dimensional and the one-dimensional characteristic manifolds of a first order partial differential equation (Duke Mathematical Journal, vol. 3 (1937), p. 740). The method is as follows: first, derive the differential equations of an $(n-1)$-dimensional characteristic manifold as a set of consistency conditions which must be satisfied over exceptional manifolds, that is, manifolds over which the determinant of the left member of a system of linear equations vanishes; regard this set of consistency conditions as a new system of differential equations to be solved; repeat the characteristic process on this system of equations to obtain characteristic sub-manifolds of $(n-2)$ dimensions. Continuing this step-by-step process we finally arrive at characteristic sub-manifolds of one dimension. These are shown to be the classical characteristic strips of Cauchy and Lie. (Received March 29, 1940.)

368. A. D. Wallace: *On separation spaces.*

The purpose of this paper is to give a new system of axioms for topological spaces in which the notion of "mutual separation" is taken as undefined. It is shown that a set of separation axioms suffice to characterize a T_1-space, but examples indicate that a weaker topology would be more natural. Separation spaces seem to be well adapted to the study of analytic topology and a version of the Whyburn-Eilenberg factor theorem is given under very general conditions and without the use of upper semi-continuous decompositions. (Received April 26, 1940.)

369. J. L. Walsh and W. E. Sewell: *Degree of polynomial approximation to analytic functions. Problem β.*

If $f(z)$ is analytic for $|z|<1$ and satisfies the inequality $|f(z)| \leqq M_1(1-|z|)^{p+\alpha}$, where p is a negative integer and $0<\alpha<1$, $f(z)$ is said to be of class $L(p, \alpha)$. If $f^{(p)}(z)$ is analytic for $|z|<1$, continuous for $|z|\leqq 1$, and satisfies a Lipschitz condition on $|z|=1$ of order α, where p is a nonnegative integer and $0<\alpha<1$, $f(z)$ is said to be of class $L(p, \alpha)$. If $f(z)$ is of class $L(p, \alpha)$, there exist polynomials of respective degrees n such that $|f(z)-p_n(z)| \leqq M\rho^{-n}n^{-p-\alpha}$ on the circle $|z|=1/\rho<1$. Reciprocally if there exist polynomials of respective degrees n such that $|f(z)-p_n(z)| \leqq M\rho^{-n}n^{-p-\alpha-1}$ on the circle $|z|=1/\rho$, then $f(z)$ is of class $L(p, \alpha)$. These results extend to approximation on an arbitrary analytic Jordan curve. (Received April 15, 1940.)

370. J. V. Wehausen: *A space isomorphic to Hilbert space.*

A Banach space is constructed which is isomorphic to Hilbert space but which contains no euclidean spaces of dimension equal to or greater than 2. It is constructed as follows. Let $x=(x_1, x_2, \cdots)$ and $x^{(k)}=(x_1, x_2, \cdots, x_k, 0, \cdots)$. Define $\|x^{(1)}\|=|x_1|$, $\|x^{(k+1)}\|^{p_k}=\|x^{(k)}\|^{p_k}+|x_{k+1}|^{p_k}$, $(p_k\geqq 1)$, and $\|x\|=\lim_{k\to\infty}\|x^{(k)}\|$. The set of sequences for which $\|x\|$ is finite is the required space if p_k converges to 2 sufficiently fast ($p_k=2-1/k$ is adequate). (Received April 10, 1940.)

371. G. T. Whyburn: *On multicoherence.*

Consider the number $r(X)$ defined by Eilenberg for continua X as l.u.b. $[r(X_1, X_2)]$, where $X=X_1+X_2$ is any decomposition of X into continua X_1 and X_2 and $r(X_1, X_2)+1$ is the number of components of $X_1\cdot X_2$. It is shown that this number is cyclicly additive, that is, $r(M)=\sum r(E_i)$, where M is a locally connected continuum with true cyclic elements $[E_i]$. Also a new and direct proof (based on the simple fact that if a sum of k connected sets is connected, a connected partial sum consisting of any given number less than or equal to k of terms can be found) is given for the result due essentially to Eilenberg that if the transformation $f(A)=B$ is quasi-monotone in the sense of Wallace (that is, for each continuum K in B with a non-vacuous interior, $f^{-1}(K)$ consists of a finite number of components each mapping onto K), $r(B)\leqq r(A)$. This gives $r(B)\leqq r(A)$ for monotone transformations on continua A and for interior transformations on locally connected continua A. (Received April 27, 1940.)

372. G. T. Whyburn: *On singularities and inversion under interior transformations.*

A subset K is said to locally separate a set M at $x\in K$ provided that for every sufficiently small neighborhood V of x in M, $V-V\cdot K$ is disconnected. It is shown that if A is locally compact, $f(A)=B$ is continuous and interior on $A-E$ where $E\subset A$, x is any point of $E\cdot\overline{A-E}$ which is a component of $f^{-1}f(x)$, and U is any neighborhood of x, then $f(E\cdot U)$ locally separates B at $f(x)$. Thus, in particular, if E is not open in A and $f(E)$ does not locally separate B at any point, f is interior on A. (This includes and extends results of Stoilow and the author on singularities of interior transformations.) It is also shown that, if A is a locally connected continuum and $f(A)=B$ is interior and light, (1) if Y is a locally connected continuum in B whose interior is dense in Y, $f^{-1}(Y)$ is locally connected, (2) if A is on a 2-manifold, and p is any point of B of order greater than 2, $f^{-1}(p)$ is a finite set of points. These two results form the basis for a greatly simplified proof for the theorem that, under the conditions of (2), if K is a locally connected continuum in B, $f^{-1}(K)$ is locally connected. (Received April 27, 1940.)

373. G. T. Whyburn: *Transformations on boundary curves.*

For any subset $\underline{X}$ of a continuum M let $\alpha(\underline{X})$ denote the number of components of $M-\underline{X}$. If A is a boundary curve (equal to a Peano continuum every true cyclic element of which is a simple closed curve) and $\underline{P}=p_1+p_2+\cdots+p_m$ is any finite subset of M, it is shown that $\alpha(\underline{P})\leqq\alpha(p_1)+\cdots+\alpha(p_m)$ and the equality holds if, for each i and j, $\underline{P}$ contains all points of A separating p_i and p_j. This is used to prove that if $f(A)=B$ is non-alternating and light, for each $y\in B$, $\alpha(y)=\sum\alpha(x)$, $x\in f^{-1}(y)$. The converse holds if B is assumed to be a boundary curve or if $\alpha(y)$ is assumed finite for every $y\in B$. It results from this that if A, B and C are boundary curves and the transformations $f_1(A)=B$ and $f_2(B)=C$ are non-alternating and light, so also is the composite transformation $f(A)=f_2f_1(A)=C$. (Received April 27, 1940.)

374. Paul Alexandroff: *General combinatorial topology.*

The fundamental duality theorem which correlates the r-dimensional cohomology group of a closed set with the $(r+1)$-dimensional cohomology group of the residual set is here obtained by a systematic development of homology theory based on the inverse and direct spectra. The same development leads also to a simple definition of the cocycle ring. Thus the theory of the spectrum, a standard topological and group-theoretical concept, is now made to include results which have hitherto been obtained only by the special methods of Alexander and of Kolmogoroff. (Received May 8, 1940.)

375. J. P. Ballantine: *Proof of Poincaré's geometric ring theorem.*

Referring to the proof of Poincaré's geometric ring theorem (Transactions of this Society, vol. 14 (1913)) by G. D. Birkhoff, the author has attempted to supplement the reasoning given there by two lemmas. It is assumed that T is a continuous one-to-one transformation having no invariant point. The index of a closed curve C relative to T is defined as the number of revolutions advanced by the vector from P to $T(P)$ as P moves around C. Lemma 1: If there exists a circuit of index different from zero relative to T, there is one of index different from zero which can be inscribed in a circle of radius ϵ. Lemma 2: If there exists a circuit C of index different from zero inscribed in a circle of radius ϵ, and if $2\epsilon<\delta$, where δ is the least distance moved by any point under T, then the index of C is ± 1, $T(C)$ surrounds the circle of radius ϵ, and the area of $T(C)$ exceeds that of C. It follows that T is not area preserving. (Received June 24, 1940.)

376. E. F. Beckenbach: *An integral analogue of Laplace's equation.*

It is shown that if the real function $x(u, v)$ is continuous in a finite simply connected domain D then a necessary and sufficient condition that $x(u, v)$ be harmonic in D is that $\int_{C(u_0, v_0; r)} [\int_{C(u_0+s, v_0+t; \rho)} x(u_0+s+\xi, v_0+t+\eta)(d\xi+id\eta)](ds-idt)=0$ hold for all circles $C(u_0, v_0; r+\rho)$ in D. (Received April 27, 1940.)

377. R. S. Burington and J. M. Dobbie: *A new family of wing profiles.*

Piercy, Piper, and Preston (Philosophical Magazine, (7), vol. 24 (1937), pp. 425–444, [P]), using conformal mapping, have recently advanced a new family of wing profiles F which are thought to have certain advantages over earlier theoretical wing shapes. The present paper reconsiders this work from a rigorous mathematical point of view, with the purpose of definitely exhibiting a transformation which fulfills all the requirements as to analyticity and boundary conditions essential to the theory (Burington, *On the use of conformal mapping in shaping wing profiles*, to appear in the American Mathematical Monthly). Special attention is given to the singularities of the transformations used. A direct by-product of this study leads to a new and more general family B of airfoils, a subfamily of which is the family F. The sequence of transformations advanced herein, which maps a circle into an airfoil of family B (and F), seems much simpler than that used in the paper [P]. (Received May 15, 1940.)

378. J. H. Curtiss: *Necessary conditions in the theory of interpolation in the complex domain.*

Let $f(z)$ be a function which is analytic on a closed limited region R whose boundary B is the boundary of an infinite region K. Let $L_n(z;f)$ be the polynomial of degree at most $n-1$ which coincides with $f(z)$ in the points $\alpha_1^{(n)}, \alpha_2^{(n)}, \cdots, \alpha_n^{(n)}$ chosen on B. If at a single interior point z_0 of R, $\limsup_{n\to\infty} |L_n[z_0; 1/(t-z_0)]|^{1/n} \leqq 1$, all t in K, it follows that (i) $|\omega_n(z)|^{1/n} \to \Delta$ uniformly on any closed point set of the interior R, where Δ is the transfinite diameter of R and $\omega_n(z) = \prod_1^n (z-\alpha_k^{(n)})$, (ii) $L_n(z; f) \to f(z)$ uniformly, z in R, for each function $f(z)$ analytic on R, (iii) if B is a Jordan curve, the points $\alpha_k^{(n)}$ are uniformly densely distributed on B. In establishing this result, several auxiliary results are developed in some detail, including some new non-trivial sufficient conditions for convergence of the sequence $\{L_n\}$ and an inequality which promises to be useful in the study of the polynomials $\omega_n(z)$. (Received May 31, 1940.)

379. E. L. Dodd: *The probabilities for certain inequalities among the values of a moving average of chance variables.*

Let independent chance variables x_i be subject to the same law of distribution. Let $z_r = \sum x_i$, $i = r-n+1$ to r. Let p be the probability that $z_r \leqq z_{r+1} \leqq \cdots \leqq z_{r+k}$ or, more generally, that among $k+1$ consecutive z's a specified ordering should obtain. Then for every $n \geqq k$, this value of p is a constant. If, further, the x's are normally distributed, and s and t are whole numbers, the probability that $z_{r-s} \leqq z_r \geqq z_{r+t}$ is at least equal to $\frac{1}{4}$. The above statements remain valid if z_r denotes the moving average $\sum x_i/n$, instead of the moving sum. (Received May 20, 1940.)

380. H. E. Goheen: *On the primitive groups of the classes $4p$ and $5p$.*

The author proves that there are no primitive groups of class $5p$, p a prime greater than 5, which contain a permutation of degree $5p$ and order p, and that with a few exceptions there are no primitive groups of class $4p$ which contain a permutation of order p and degree $4p$. The exceptions are $p=5$, in which case every such group is contained in the holomorph of the elementary abelian group of order 25, and $4p+1=q$, q a prime or a power of 5, in which case all the known groups of the required type are primitive subgroups of the triply transitive Mathieu group. The proof is based upon the examination of the normalizer of the cyclic group generated by the permutation of order p and minimum degree in the group in the light of a theorem of W. A. Manning (this Bulletin, vol. 13 (1906), p. 201), and the examination of the subgroup fixing one letter of a simply transitive primitive group in the light of a theorem of Weiss (this Bulletin, vol. 40 (1934), p. 402). Together with auxiliary considerations these dispose of every possibility except one, which is disposed of by means of the theory of group characters. (Received May 22, 1940.)

381. O. G. Harrold (National Research Fellow): *A note on strongly irreducible maps of an interval.*

A continuous mapping of the compact metric space A onto B has been called *strongly irreducible* provided no proper closed subset of A maps onto all of B. If M is any Peano space in which the nonlocal separating points are dense and P is any given countable dense set of nonlocal separating points, there exists a continuous mapping, $f(I)=M$, where I denotes the unit interval, such that $y \in P$ implies $f^{-1}(y)$ is a single point and $\overline{f^{-1}(P)} \supset I$. The set J^* of such strongly irreducible maps is a dense G_δ in the space J of maps of I onto M. These mappings are nowhere arc preserving in the sense that the only arcs which are transformed into arcs are the individual points. (Received May 21, 1940.)

382. L. B. Hedge: *Moment problem for a bounded region.*

A solution of the moment problem for a bounded interval, given by Hausdorff in 1923, is extended to any bounded region in euclidean n-space under certain conditions on polynomial expansions over the region. The resulting solution is shown to be valid for the n-dimensional sphere, and includes the Hausdorff case as well as known conditions on the "class" of Fourier and Fourier-Stieltjes series. (Received May 17, 1940.)

383. Ralph Hull: *The structure of certain Fuchsian groups.*

The units of a maximal order of a rational, indefinite, quaternion algebra form an infinite group which may be represented as a Fuchsian group, and whose structure is determined in a simple way by the fundamental number of the algebra (cf. American Journal of Mathematics, vol. 61 (1939), pp. 365–374). The group of units of a non-maximal order of such an algebra may similarly be represented as a Fuchsian group, and analogous methods may be applied to determine its structure, that is, the genus, and class-number of elliptic substitutions in the group. Groups thus studied include those which correspond to the totality of integral solutions of equations such as $x^2+ay^2-bu^2-abv^2=1$, where a and b are positive integers. (Received May 21, 1940.)

384. Bella Manel: *Conformal mapping of multiply connected domains on the basis of Plateau's problem.*

This paper is concerned with the conformal mapping of an arbitrary multiply connected plane domain G on the following normalized domains: B_1, the half-plane with finite slits radial to the origin one of which lies on the imaginary axis; B_2, the half-plane with elliptical holes having their axes mutually parallel and having a fixed eccentricity; B_3, the unit circle with concentric circular slits one of which has end points on the real axis. The method is to solve the Plateau problem for the contour bounding G with the desired conformally equivalent domains as the domains of representation. The solution yields the analytic function which performs the mapping. This connection between conformal mapping and the Plateau problem has been stressed by Douglas and by Courant. The procedure of Courant forms the basis of this investigation. For plane contours the existence of a solution to a minimum problem involving the Dirichlet integral is used. By a detailed analysis of the variational conditions it is shown that the solution to this minimum problem is a minimal surface for each of the normal domains B_1, B_2, B_3. (Received May 27, 1940.)

385. A. F. Moursund: *A note on Gibbs' phenomenon.*

The kernel for Nevanlinna's weak summation method, which is essentially the same as the Bosanquet-Linfoot zero order method, contains a constant whose value has no effect in determining the strength of the method in forcing convergence. In this note it is shown that this constant plays an important role in determining the power of the method to destroy Gibbs' phenomenon in Fourier series. Weak methods are exhibited which destroy Gibbs' phenomenon while stronger methods do not. (Received May 22, 1940.)

386. T. S. Peterson: *The Fredholm minors for Goursat's kernel.*

The purpose of this note is to give the explicit determinantal form for the Fredholm minors of a kernel having the form $\sum_{i=1}^{n} M_i(x)N_i(y)$. The result is obtained directly using theorems due to Platrier and Sylvester. (Received May 21, 1940.)

387. R. M. Robinson: *On the approximation of irrational numbers by fractions with odd or even terms.*

All fractions may be divided into four types, according as the numerator and denominator are odd or even. Let one or more of these types be selected as admissible. Using continued fractions, the largest value of μ is determined for which to every irrational ξ infinitely many admissible fractions A/B can be found satisfying $|\xi - A/B| < 1/\mu B^2$. A part of this problem was solved by W. T. Scott in the February issue of this Bulletin, using a different method. (Received May 29, 1940.)

388. P. M. Swingle: *Local connectedness and biconnected sets.*

In this paper it is proved for certain spaces that if a biconnected set B is connected im kleinen at a point p, then p is a dispersion point of B. The problem is generalized in several ways. (Received May 17, 1940.)

389. P. M. Swingle: *Non-transfinite connected sets.*

In this paper the definitions of finitely-containing, finitely-divisible, and finitely-convergeable connected sets are modified, substituting in place of connected subset either biconnected, widely connected, or punctiform connected subset. The relations between these new types of sets are studied, a solution being obtained of previously proposed problems (American Journal of Mathematics, vol. 53, pp. 373, 400, Problems 3 and last half 11). Also certain results are obtained concerning a modular connectivity which is defined. (Received May 17, 1940.)

390. C. W. Vickery: *On cyclically invariant graduation.* II.

The author has extended the results of an earlier paper (abstract 46-5-336, April 27, 1940). Suppose that $g(t)$ is a periodic function of period $2k$ having an expansion in Fourier series. Suppose that X is a random variable with characteristic function $f(w)$ such that, for every integer $n, f(n\pi/k) \neq 0$ and is an even function. Let $g^*(t) = g(t) - a_0/2$, where $a_0 = (1/k)\int_{-k}^{k} g(t)dt$. For every integer $m \geqq 0$, graduation operator G_m is defined as follows: $G_m\{g(t)\} = a_0/2 + E\{g^*(t+X)\}/f(m\pi/k)$. The terms of index m of the Fourier expansion of $g(t)$ are invariant with respect to the application of G_m. Necessary and sufficient conditions have been found that the application of G_m to any Fourier series produce a Fourier series. Sufficient conditions have been found that the application of G_m to any Fourier series produce a Fourier series that is convergent almost everywhere. (Received April 23, 1940.)

391. C. W. Vickery: *On systems of Fourier coefficients.*

If $\sum (a_n \cos nx + b_n \sin nx)$ is a Fourier series and $[\lambda_n]$ is a sequence of real numbers, then in order that $\sum \lambda_n(a_n \cos nx + b_n \sin nx)$ be a Fourier series it is necessary and sufficient that there exist a periodic function $g(x)$ of bounded variation and of period 2π such that, for each integer m, $f(m) = 1/\pi \int_{-\pi}^{\pi} e^{imx} dg(x)$ is an even function and such that $\lambda_n = f(n)$, n being positive. An example shows that it is not necessary that $g(x)$ be an odd function. If, furthermore, the second derivative of $g(x)$ exists for all values of x between $-\pi$ and π and is absolutely integrable, then $\sum \lambda_n(a_n \cos nx + b_n \sin nx)$ is a Fourier series which converges for all values of x except a set of measure 0. (Received April 23, 1940.)

392. H. S. Wall: *A continued fraction related to partition formulas.*

The author shows how four well known identities involving $\prod(1-ax^n)$ can be derived from a single continued fraction, namely, $1+rx/1+(1-r)wx/1+(1-w)wrx/1+(1-wr)w^2x/1+(1-w^2)w^2rx/1+\cdots$. (Received April 6, 1940.)

393. Y. K. Wong: *On biorthogonal matrices.*

Consider the basis $\mathfrak{A}$, $\mathfrak{P}^1$, $\mathfrak{P}^2$, ϵ^1, ϵ^2 (E. H. Moore, *General Analysis*, Part I, p. 16). Suppose that κ^{12} is by columns of $\mathfrak{M}(\epsilon^1)$. In a paper entitled *On non-modular matrices*, the author establishes the one-to-one correspondence between the class $\mathfrak{M}(\epsilon_\kappa^1\ \kappa^*\ \epsilon^2)$ of all vectors μ^1 modular as to ϵ^1 such that $J^1\kappa^{*21}\mu^1$ are in $\mathfrak{M}(\epsilon^2)$ and the class $\mathfrak{M}(\epsilon^2 \cap \epsilon_\kappa^2)$ of vectors μ^2 modular as to ϵ^2 and ϵ_κ^2. In the present paper, the author first obtains some properties on a pair of matrices κ^{12}, ψ^{21}, which are by columns of $\mathfrak{M}(\epsilon^1)$, $\mathfrak{M}(\epsilon^2)$ respectively, such that $J^2\kappa^{*12}\mu^2 = J^2\psi^{*12}\mu^2$ for every μ^2 in $\mathfrak{M}(\epsilon^2 \cap \epsilon_\kappa^2)$. Next, it is assumed that ϵ_0^1, ϵ_1^1 are idempotent as to ϵ^1, κ^{12} is by columns of $\mathfrak{M}(\epsilon_0^1)$, and ϕ^{21} is by rows conjugate of $\mathfrak{M}(\epsilon_1^1)$. Take any pair of vectors μ^1, ν^1 in $\mathfrak{M}(\epsilon_0^1\ \kappa^*\ \epsilon^2)$, $\mathfrak{M}(\epsilon_1^1\ \phi\ \epsilon^2)$ respectively; if $J^1\bar{\mu}^1\nu^1$ is equal to $J^2(J^1\bar{\mu}^1\kappa^{12}, J^1\phi^{21}\nu^1)$, then ϕ^{21}, κ^{12} are said to be biorthogonal as to $\epsilon^2\epsilon_1^1\epsilon_0^1$. The purpose of the paper is to investigate the properties of the biorthogonal pairs of matrices. (Received May 22, 1940.)