treated with skill and clarity. It should be added that he will also find a short readable proof of the Cauchy formula $\int_J f(z) = 0$ in which $f(z)$ is assumed to be regular over the inner domain D of the rectifiable simple closed curve J and continuous in $D+J$. Here again the power of Alexander's lemma shows itself in solving rapidly the separation problems that arise.

P. A. SMITH

Étude Critique de la Notion de Collectif. By Jean Ville. (Monographies des Probabilités, no. 3.) Paris, Gauthier-Villars, 1939. 144 pp.

Professor Ville has written an interesting and valuable discussion of the concept of a collective, upon which many mathematicians found the theory of probability. The author discusses systems of play in detail, and generalizes this idea to that of a "martingale." This leads to a new criterion for the exclusion of sequences from probability discussions, that is, to a new definition of collective. Any given set of sequences of probability 0 can be excluded by this new criterion, whereas the system criterion, used by Copeland, Popper, Reichenbach, Tornier, Wald, can be used only to exclude certain sets of sequences (necessarily of probability 0). Ville extends the definition of a martingale to the case of a stochastic process depending on a continuous parameter, and shows that some of his sequence results go over.

It is unfortunate that this book, which contains much material which clarifies the subject, should contain so much careless writing. This ranges from uniformly incorrect page references to mathematical errors. Thus (p. 46) it is claimed (and used in a proof) that every denumerable set is a G_δ. The author's main theorem on systems is not as strong as earlier results with which he is apparently unfamiliar. (Cf. Z. W. Birnbaum, J. Schreier, Studia Mathematica, vol. 4 (1933), pp. 85–89; J. L. Doob, Annals of Mathematics, (2), vol. 37 (1936), pp. 363–367.) His discussion of random functions is inadequate and obscure, for example, his demonstration that his main theorem on martingales does not go over to the continuous process uses as an example a measure on function space not in accordance with the usual definition of probability measures on this space.

A specialist who can overlook such slips will find many stimulating ideas in this book. Other readers can profit by the comparative analysis of the different criteria for collectives, and by the discussion of martingales.

J. L. DOOB

Problems in Mechanics. By G. B. Karelitz, J. Ormondroyd, and J. M. Garrelts. New York, Macmillan, 1939. 9+271 p.

This is a collection of nearly 800 problems in statics, kinematics and dynamics. Some two thirds are based on those compiled by the late I. V. Mestchersky, of the Polytechnic Institute of St. Petersburg. The authors have not only translated these, but have replaced the metric by English engineering units and given them a background suitable to American students.

The book is intended to supplement a first course in mechanics as applied to engineering. Thus the problems vary from simple exercises in resolution of forces and falling bodies to those on tensions in cables and curvilinear motion under central forces. Many will provide hard practice in the application of mechanical principles, but none are of the puzzle type. Only rudimentary calculus or differential equations and no knowledge of Lagrange's equations is assumed. As is the case in actual engineering practice, with few exceptions the problems are reducible to those in one or two dimensions. The setting in most cases will appeal to the student as interesting and useful.

A fifty page résumé of the principles of mechanics is added. This facilitates references in the solutions which are given to some typical problems. Answers are given in practically all cases.

The book may be recommended to teachers who wish more problems than those found in the usual textbook in elementary applied mechanics.

PHILIP FRANKLIN

Elementary Matrices and some Applications to Dynamics and Differential Equations. By R. A. Frazer, W. J. Duncan, and A. R. Collar. Cambridge, University Press, 1938. 16+416 pp.

The authors are primarily interested in developing the theory of matrices as a tool, especially to be applied to aerodynamics. Consequently it is only the first four chapters which are concerned with matrices as such, while the remaining nine chapters are devoted to applications. Numerous examples occur throughout the work and much stress is put on the technique of numerical computation. An ele mentary knowledge of determinants is assumed at the outset.

In the first three chapters are found very satisfactory treatments of most of the standard theorems on matrices, including such topics as polynomials and power series of matrices, the Cayley-Hamilton theorem, Sylvester's theorem, and canonical forms for matrices. No treatment is given, however, of the reduction of a matrix to canonical form under collineatory transformations, although the fundamental results are stated and the reader is referred to a proper place in the literature. The fourth chapter is devoted to miscellaneous numerical methods for finding the reciprocal, high powers, and latent roots of a matrix, together with a few applications.

Chapters five and six treat systems of linear ordinary differential equations with constant coefficients. The fact that matrices can be used in treating systems of this kind is indubitably the reason that matrices are so important to the aeronautical engineer. The treatment given is unsatisfactory to the mathematician, because it is not shown that a complete set of solutions is given by the methods described (although this is indeed the fact); nor is there an adequate reference to fill the gap. Yet this very matter constitutes the most difficult part of the whole theory (cf. p. 168).

Chapter seven describes miscellaneous applications to linear differential equations with variable coefficients.

The rest of the book is primarily concerned with dynamics and becomes more and more technically involved in aeronautical problems as the book progresses to its close, the last chapter being entitled "pitching oscillations of a frictionally constrained aerofoil." It should be remembered, however, that much of this technicality is that of language only. On page 267, for example, we find a section entitled "the equations of motion of an aeroplane." The authors then go on to say that they are deriving the equations of motion for any rigid body, which merely for definiteness is supposed to be a (rigid) aeroplane in flight!

The book will probably prove invaluable to the student of aerodynamics. It should also be a stimulating reference book for the really good undergraduate who is assailed by doubts as to the practical applications of mathematics.

D. C. LEWIS