upon four lectures delivered at Lucknow University, he gives a synopsis of the work done in this field. The first lecture contains a brief history of the earlier attempts to construct nondifferentiable functions leading up to the example of Weierstrass. He discusses the method of Dini and the various series definitions, including the recent one due to van der Waerden. In the second lecture he treats functions defined geometrically, in particular, the curves of Bolzano, von Koch, Peano, Hilbert, Kaufmann, and Besicovitch, whereas the general method of Knopp is only mentioned. The third lecture contains his own arithmetical definitions which attach in part to earlier work of Peano and E. H. Moore. The last lecture deals with various properties of the derived numbers of nondifferentiable functions. In closing, let me add that anybody who is giving a course in real variables will find this little tract useful.

EINAR HILLE

Moderna Teoria delle Funzioni di Variabile Reale. Part 2. *Sviluppi in Serie di Funzioni Ortogonali*. By G. Vitali and G. Sansone. (Monografie di Matematica Applicata per cura del Consiglio Nazionale delle Ricerche.) Bologna, Zanichelli, 1935. 6+310 pp.

This second volume is a very valuable continuation of the first part which was reviewed in vol. 43 (1936), p. 15, of this Bulletin. It is devoted to expansions in orthogonal series and takes up quadratically integrable functions, Fourier series, Legendre series, Laguerre and Hermite series, and the Stieltjes integral. The first chapter gives the primary notions on Hilbert space, orthogonality, linear independence, approximation, convergence in the mean, and expansions in orthogonal series including the closure theorem of Vitali which serves as the basis for the discussion in the special cases. This is followed by a discussion of Fourier series, including convergence in the mean, local convergence criteria, Fejér and Poisson summability, and the Fourier integral (Fourier transforms are just mentioned). The treatment of Legendre series starts with an adequate discussion of the basic properties of the Legendre polynomials through the asymptotic formulas and leads up to Hobson's convergence theorem. The fourth chapter, dealing with Laguerre and Hermite series, is perhaps the most valuable in the whole book since these series are normally not discussed in standard texts on analysis. It presents the fundamental properties of the polynomials, including their asymptotic behavior, and ends with the convergence theorems of Stone and of Uspensky for Hermite series. The last chapter, which has very little contact with the rest of the book, gives a discussion of the Stieltjes integral with applications to the theory of distribution functions and their characteristic functions. There is a large bibliography. The treatment is up-to-date, rigorous without being heavy, and the book can be strongly recommended to those who have to give courses in real variables or classical mathematical physics.

EINAR HILLE

Summable Series and Convergence Factors. By C. N. Moore. (American Mathematical Society Colloquium Publications, vol. 22.) New York, American Mathematical Society, 1938. 6+105 pp.

Fairly early in the development of the theory of summability of divergent series, the concept of convergence factors was recognized as of fundamental importance in the subject. One of the pioneers in this field was C. N. Moore, the author of the book under review. He first introduced the name "convergence factors" in this connection, published some of the first convergence factor theorems, and has been one of the chief investigators in the subject since then. It is therefore appropriate that the first systematic treatment of convergence factor theory in book form should have been written by him.

The author points out that all methods for the summation of a divergent series which have come into general use may be classified as mean-value methods or convergence-factor methods. In either case the object of the summability method is to determine a value or "generalized sum" for the series. Although convergence factors were first applied to summable series, it was soon seen that they could yield valuable information when applied to convergent series.

Moore classifies convergence factors into two types. In type I he places the factors which have only the property that they preserve convergence for a convergent series or produce convergence for a summable series. In type II he places the factors which not only maintain or produce convergence but have the additional property that they may be used to obtain the sum or generalized sum of the series. This book gives a generalized systematic treatment of the theory of convergence factors of both types, for simply infinite series and for multiple series, convergent and summable. For summability the author uses the method of Nörlund means instead of the Cesàro method, giving more general results. Many of the theorems and methods given in this work are original and the proofs have not been published elsewhere.

The book opens with an Introduction in which the early history of the idea of summability of series is discussed, and the emergence of the concept of convergence factors is traced historically. Chapter 1 deals with convergence factors in convergent series. For convergence factors of type I, necessary and sufficient conditions are derived in order that a convergent series may still converge after convergence factors are introduced into its terms. For factors of type II, additional necessary and sufficient conditions are obtained in order that the series with convergence factors may converge to the sum of the original series. Corresponding results are derived for multiple series whose partial sums are bounded. These convergence factor theorems serve to furnish criteria for the regularity of convergence-factor definitions and mean-value definitions of summability; they may also be used to obtain results concerning relations of equivalence and inclusion of summability methods.

In Chapter 2, a brief discussion is given of Nörlund summability of series, simple and multiple. Chapter 3 then takes up, for summable simple series, the corresponding questions to those of Chapter 1, for convergence factors of both types; Chapter 4 handles the case of summable double series, and Chapter 5 that of summable multiple series of higher order. Results for the Cesàro method of summability are obtained as special cases. The final chapter is concerned with convergence factors in restrictedly convergent multiple series. The book closes with a rather full bibliography of the subject.

Many of the convergence factor theorems hitherto published in the literature appear as special cases of the general theorems here derived. This book makes considerable progress in the attempt to present a definitive form to the subject of convergence factors.

L. L. SMAIL

Analyse Mathématique. Vol. 1. *Analyse des Courbes, Surfaces et Fonctions usuelles. Intégrales simples.* By Paul Appell. 5th edition, completely revised by Georges Valiron. Paris, Gauthier-Villars, 1937. 8+395 pp.

The first edition of this work appeared in 1898 and, according to the preface, it reproduced the course which for three years Paul Appell had been offering at l'École Central des Arts et Manufactures. It covered the essential elements of mathematical