

forms and of more general Fourier-Stieltjes transforms (and also of related Laplace, Mellin, Hankel, Watson, . . . , transforms) on the one hand is becoming an almost universal tool for treating various problems arising in the theory of functions of a complex variable, theory of linear operators, harmonic analysis, probabilities, mathematical physics; on the other hand it offers an inexhaustible source of formulas and relations which are interesting, or at least curious, in themselves, irrespective of possible applications. The present monograph, which hardly can be considered as an "introduction," centers its attention mainly in the latter aspect of the theory of Fourier integrals and presents a wealth of interesting material, a considerable portion of which is due to most recent investigations. A partial list of contents follows.

Chapter I (Convergence and summability, pp. 1-49) treats of formal aspects of Fourier, Laplace, and Mellin transforms, and gives fundamental results concerning convergence and summability (Cesáro, Cauchy, Poisson, Weierstrass) of the corresponding integrals. Chapter II (Auxiliary formulae, pp. 50-68) gives a preliminary survey of the Parseval formula, theory of convolution (Faltung) of Fourier transforms, and Poisson summation formula. Chapter III (Transforms of the class L^2 , pp. 69-95) is devoted to a treatment of Fourier transforms in L^2 together with some related topics. Results of this chapter are partially extended to transforms in L^p in Chapter IV (Transforms of other L -classes, pp. 96-118). The theory of conjugate trigonometric integrals and the closely related theory of Hilbert transforms is dealt with in Chapter V (Conjugate integrals, Hilbert transforms, pp. 119-151), while the next, Chapter VI (Uniqueness and miscellaneous theorems, pp. 152-176), in addition to the problem of unique representation, treats of some refinements of the Parseval formula and problems of growth of Fourier transforms. Chapter VII (Examples and applications, pp. 177-211) contains a considerable number of special formulas involving Fourier integrals. The theory of "general transforms," which was originated recently by Watson, and the theory of "self-reciprocal" functions with their various generalizations are discussed in Chapter VIII (General transforms, pp. 212-244) and Chapter IX (Self-reciprocal functions, pp. 245-274). Next, Chapter X (Differential and difference equations, pp. 275-302) contains numerous special applications to the theory of difference and differential equations, giving a rigorous exposition of some parts of the theory known under the name of "operational calculus." The last chapter, XI (Integral equations, pp. 303-369), deals with a considerable number of special integral equations. The book closes with a substantial bibliography (pp. 370-387) and a short index.

J. D. TAMARKIN

Methoden und Probleme der dynamischen Meteorologie. By H. Ertel. (Ergebnisse der Mathematik und Ihrer Grenzgebiete, vol. 5, no. 3.) Berlin, Springer, 1938. 4+122 pp.

The mathematician will perhaps be surprised to learn that the difficulties in the study of the dynamics of our atmosphere are essentially of a mathematical nature. In fact this subject, as well as stellar hydrodynamics, offers a virgin field for the applied mathematician, and it is to be hoped that Ertel's monograph will serve to attract the mathematical skill which meteorology needs.

As can be inferred from the title, Ertel's monograph is not intended to serve as a textbook in meteorology. One important omission is atmospheric turbulence. The problems that are treated have mostly been the subject of Ertel's own researches and here, as in the original papers, the elegance of the treatment may seem a bit luxurious to the practical meteorologist.

A feature which is not found in textbooks is the formulation of the variational

principle in the dynamics of the atmosphere. It is also gratifying to find at last in meteorological literature the name of Kelvin mentioned (p. 58) in connection with the problem of convective instability of moist air, which he treated for the first time in 1865. In his treatment of the heat balance of the atmosphere the author fails to stress a fundamental difficulty, which is our ignorance, at present, of the radiative behaviour of water vapor under atmospheric conditions. The result of Hergessel referred to on page 76, that in a semi-grey atmosphere at radiative equilibrium there would exist a uniform temperature of -54°C , has been shown to be incorrect.*

On the whole the book offers a convenient exposition of present-day problems of dynamic meteorology.

C. L. PEKERIS

Technique de la Méthode des Moindres Carrés. By Henri Mineur. (Monographies des Probabilités, publiées sous la direction de M. Émile Borel, no. 2.) Paris, Gauthier-Villars, 1938. 8+93 pp.

Mineur states in the preface of his book that it should be possible for the reader to learn to use the method of least squares without understanding the theory behind it. In fact, the theory of the various operations is not fully explained until the fifth chapter. For this reason the mathematician will perhaps find it more satisfactory to proceed directly from the first chapter to the fifth and then read the second, third, and fourth. We shall follow this order in discussing the topics which Mineur treats.

The first chapter consists of an example for which it is desired to fit a linear equation to a set of data. The attempt to make such an equation conform exactly leads in general to an incompatible system. The equation which is the best fit in the sense of least squares is shown in Chapter 5 to result from the so-called normal equations which are compatible and linear. The resolution of a linear system by means of determinants is cumbersome and hence the author presents the Gaussian method which constitutes the principal part of the technique. The method of least squares is shown to be equivalent to finding the equation which produces the minimum probable error. Also in Chapter 5 one finds a discussion of such concepts as mean, standard deviation, probable error of a single measurement, and probable error of the mean of a set of measurements. In fact the author gives a brief but clear exposition of the elements of statistics. It is, however, surprising that simple, multiple, and partial correlation are omitted.

Chapter 2 contains a detailed description of the method of tabulating the data, forming the normal equations, and solving them. Mineur fails to note that much of this tabulation is unnecessary when a computing machine is used. For example, by means of a machine it is possible to obtain a sum of products as a single operation without recording the individual products. In Chapter 3 the author discusses the nature of errors of measurement. He also indicates how the method of least squares can be applied to nonlinear equations. In Chapter 4 he applies his method to the solution of a problem in stellar statistics.

On the whole, this is a readable and useful book.

A. H. COPELAND

British Association for the Advancement of Science: Mathematical Tables. Volume 6: *Bessel Functions.* Part 1: *Functions of Orders Zero and Unity.* Cambridge, University Press; New York, Macmillan, 1937. 20+288 pp.

The preface to this volume opens with the words: "It is with the satisfaction of

* C. L. Pekeris, *Gerlands Beiträge*, vol. 28 (1930), p. 377.