ticians in projective differential line geometry. With this end in view the author makes no attempt to give a complete presentation of any part of the subject, choosing rather to introduce the reader to several of the important phases of line geometry. These phases have been selected on the basis of their simplicity and their geometric content. The treatment is necessarily analytical, but the geometric significance of various theorems is illustrated by interpreting them in terms of the properties of certain discrete systems of lines. However, no mention is made of the representation of lines by the points of a quadric hypersurface in projective 5-space; the reviewer believes that the use of this representation would have added interesting geometric content to many of the theorems, especially in the earlier chapters.

In the first chapter the author presents the fundamental notions of projective line geometry. Two types of line coordinates are used, projective (Plücker) coordinates and vector coordinates for treating problems in euclidean space. The effect of a point transformation on the line coordinates is studied and applied to the classification of linear complexes and related topics. Chapter 2 is concerned with one parameter systems of lines, or ruled surfaces. The treatment is purely projective and parallels that usually given for space curves. Invariants are found and are shown to determine the surface to within a projective transformation. There is also a discussion of ruled surfaces invariant under a group of transformations. Chapters 3 and 6 discuss two and three-parameter systems in a similar manner. Tensors are used to determine the invariants of the systems. Chapter 4 considers some special two-parameter systems, including those invariant under a group of transformations. In Chapter 5, vector line coordinates and the theory of two-parameter systems are applied to the theory of infinitesimal deformations of surfaces.

R. J. Walker

Principles of Mathematics. By Bertrand Russell. 2d edition. New York, Norton, 1938. 39+534 pp.

The first edition of this book is so well known, being the author's first important publication on mathematical logic, that any description here would be superfluous. The text is a reprint of the 1903 edition with, however, an interesting new introduction by the author discussing the developments in mathematical logic since it was first published. As he says "such interest as the book now possesses is historical" since the same ground was subsequently traversed with far more rigor in the *Principia Mathematica.*

In dealing with later developments the author discusses the three current views of the nature of logic and mathematics, the logistic (his own view), the formalist, and the so-called intuitionist. He sums up the discussion by saying that modern developments have resulted in an outlook "less Platonic, or less realistic in the medieval sense of the word"; that is, the view that the principles of logic, which we seem to find at different stages, are real and absolute, has receded. Put otherwise, Wittgenstein's view that what logic is "in itself" (whatever this means) "cannot be said but can only be shown" has been confirmed. Any attempt to "say" it can only be one exemplification out of many possible, not a uniquely true statement. The three current views differ, it is true, as to whether certain specific propositions are true or false, but, if Wittgenstein is correct, the removal of these differences would not result in showing that one of these views was true and the other two false, but would leave all three as alternative ideologies by which logic could be symbolized. The choice between them would be aesthetic, or based on convenience, depending on which brings into clear-cut relief some aspect of the matter in which we are interested.

The author's Introduction is not devoid of traces of an aesthetic standpoint. Discussing Carnap's *Logical Syntax of Language* containing two logical languages, one with, and the other without, the multiplicative axiom and the axiom of infinity, he says "I cannot myself regard such a matter as one to be decided by our arbitrary choice. It seems to me that these axioms either do, or do not, have the characteristic of formal truth. I confess, however, that I am unable to give any clear account of what is meant by saying that a proposition is true in virtue of its form."

A one-to-one correspondence can easily be established between these remarks and the well known couplet,

"I do not like thee, Dr. Fell
The reason why I cannot tell."

Of course, there is nothing surprising about this, as the impulse to many mathematical developments lies in similar obscure semi-aesthetic beginnings.

FREDERICK CREEDY

Integralgeometrie, I. By W. Blaschke. Paris, Hermann, 1935. 22 pp.

Integralgeometrie, V. By L. A. Santalo. Paris, Hermann, 1936. 54 pp.

Vorlesungen über Integralgeometrie. Vol. 1. By W. Blaschke. 2d edition. Leipzig and Berlin, Teubner, 1936. 60 pp.

Vorlesungen über Integralgeometrie. Vol. 2. By W. Blaschke. Leipzig and Berlin, Teubner, 1937. 68 pp.

Über eine geometrische Frage von Euclid bis Heute. By W. Blaschke. Leipzig and Berlin, Teubner, 1938. 20 pp.

The first two of these pamphlets are numbers 252 and 357 of the Actualités Scientifiques et Industrielles, and form the first two volumes of a series entitled *Exposés de Géométrie*, published under the direction of Wilhelm Blaschke. The last three pamphlets are volumes 20, 22, and 23 of the Hamburger Mathematische Einzelschriften.

The subject of integral geometry, devised for application to problems in geometric probability (such as Buffon's "needle problem"), has, under the guidance of Blaschke and his students, become an elegant theory of integral invariants, with applications not only to geometric probability, but also to differential geometry, maximum and minimum problems, and geometrical optics.

The first pamphlet is an exposition of the foundations of the subject. A "density" is defined for linear subspaces E_r of euclidean n-space E_n; this density is a differential form in the coördinates of the space S of r-dimensional linear subspaces of E_n. It has the following invariance properties: (1) invariance (Parameterinvarianz) under a change of coördinates in S; (2) invariance (Bewegungsinvarianz) under motions of E_n into itself. A "kinematic density" is also defined; it is essentially a density for rectangular coördinate systems C in E_n. In addition to the previous two invariance properties, it is unchanged (Wahlinvarianz) if each coördinate system C is replaced by one rigidly joined to it. This makes it ideal as a density for the positions of a solid body in E_n. These densities are (except for a constant factor) uniquely determined by their invariance properties. Upon integration of them, one obtains an invariant "measure" for linear subspaces of E_n, and an invariant "kinematic measure." Similar measures can be defined in spherical and non-euclidean n-spaces.

The second pamphlet is an original study of the kinematic measure in E_3 with applications. Probably the most important and typical example of the results is the following. Let K be any convex body in E_3; denote its volume by V, the area of its