

A very welcome addition is (Theorem 61) Witt's proof of Wedderburn's theorem that every finite division algebra is commutative. As given here the proof is almost too concise and it is perhaps unfortunate that it is not mentioned that the proof depends on Theorem 62. Section 43, formerly "Automorphisms of Abelian Groups," has been enlarged to include automorphisms of  $p$ -groups by the addition of Theorem 115, the Burnside basis theorem, and Theorem 116 by P. Hall on the order of the group of automorphisms of a  $p$ -group. In Section 47 the theorem of Burnside that a Sylow subgroup which is in the center of its normalizer is a factor group is proved by constructing the homomorphism. It is proved that a group whose Sylow subgroups are cyclic must be solvable. (The earlier editions proved this merely for groups of square-free order.)

In Chapter 12 on Characters, he adds a section (§60) giving an excellent exposition of the relation of representation theory to the theory of semi-simple algebras, and another (§61) giving Weyl's representations of the symmetric group by means of the Young symmetry operators. Section 65 now gives Witt's new proof of the theorem of Frobenius that a transitive permutation group in which every element except the identity displaces  $n$  or  $n-1$  letters has a normal subgroup consisting of the identity and the  $n-1$  elements displacing  $n$  letters.

Chapter 16 is all new and treats the composition and invariants of the complete linear homogeneous group.

A collation with the second edition reveals the further additions and changes:

Page 12. A non-redundant statement of the group axioms.

Page 27. A discussion of generating relations for the group of inversions and translations.

Page 92. The former Figure 34 has been replaced by Figure 37, a design from the grave of Senmut in the necropolis at Thebes.

Schluss. Inclusion of references to Deuring's *Algebren* and van der Waerden's *Moderne Algebra*.

Namenverzeichnis. Inclusion of twelve new names. The reference to Faà di Bruno should read p. 230 rather than p. 239.

Sachverzeichnis. Inclusion of seven new terms, in particular "Gruppoid" and "Idempotent."

MARSHALL HALL

*Differential Systems*. By J. M. Thomas. (American Mathematical Society Colloquium Publications, vol. 21.) New York, American Mathematical Society, 1937. 9+119 pp.

There are two types of "differential systems," systems of (partial) differential equations and pfaffian systems. It has been known since Cauchy and Pfaff that there are many relations between solutions of the corresponding two types of equations. It is the main purpose of the present book to develop the existence of solutions for the two types of equations from a formalized algebraic approach, and to exhibit their relations from the view point of Riquier's theory of "orthonomic" systems.

After two introductory chapters, Chapter III presents the formal theory of Grassmann algebra and Chapter IV the transformation of pfaffian systems into canonical form. Chapter III is self-contained and includes an elegant theory of determinants and skew-symmetric matrices. In Chapter IV the properties of the underlying ring of differentiable functions which appear as coefficients are very general, and are formally enunciated as *differential and integral assumptions*. The main assumption is that a pfaffian system in  $n$  "marks" of rank  $n-1$  has a differential basis, and it is implied

by the assumption that an algebraic system of partial differential equations which, corresponding to each unknown, contains at most one equation with a derivative of that unknown for leader (a so-called *normal* system) has a unique solution for each set of initial determinations.

Chapters V and VI, on which later chapters are modeled, are devoted to the study of solutions of systems of polynomial equations and inequations in variables  $y_1, \dots, y_n$ . The treatment is simplified to a surprising extent by the admission of the inequation on equal footing with the equation, and it leads to a decomposition of every system into a finite number of canonical systems without common roots. A canonical system embraces a sequence of at most  $n$  polynomials, each of which contains variables  $y_k$  not occurring effectively in the preceding ones, and its solvability is almost trivial. But it should be pointed out that the author does not try out his treatment on more elaborate questions concerning multiplicity of roots and ideals of polynomials. In Chapters VII and VIII the author presents his decomposition of algebraic systems of partial differential equations into *passive standard* systems. These are systems which, in addition to being canonical (as in Chapter VI), satisfy a condition of "integrability"; and their solution can in turn be made to depend on the successive solution of a finite number of normal systems. The treatment of normal systems in the author's version of Riquier's existence theorem is completed in Chapter X. But the book does not include topics of the type of Ritt's extension of Hilbert's "Nullstellensatz" to differential systems.

In Chapter IX the investigations of the previous chapters are applied to a study, with generalizations of some results to non-linear forms, of Cartan's existence theorems for pfafrican systems. For instance, the integral varieties of a pfafrican system satisfy a related system of partial differential equations, and the solutions of its principal canonical factors are all those *non-singular* integral varieties whose dimension is equal to the genus of the pfafrican system. Finally, Chapter XI gives several examples to illustrate reduction of pfafrican forms, Riquier's dissection of a Taylor series corresponding to a system of monomials, and reduction of polynomial and differential systems.

SALOMON BOCHNER

*Integralgleichungen.* By G. Hamel. Berlin, Springer, 1937. 8+166 pp.

This book has grown out of a series of lectures delivered in the spring of 1937 at the Extension Institute of the Technische Hochschule in Berlin. The lectures are addressed to men in practical work, with the general purpose of presenting topics not well known to them and of indicating applications of the theory discussed. The success of the lectures on integral equations suggested the desirability of publishing them for the benefit particularly of engineers and physicists.

The book contains no new results of interest to the mathematician and could be used as a textbook only when supplemented by references to original sources and other standard works. The author has, however, achieved considerable success in presenting the fundamental concepts and lines of argument against a background of ideas based upon definite physical problems.

The first part of the book (91 pages) contains the material as presented in the lectures. The different standard types of integral equations are introduced by the familiar problems associated with a vibrating string, and their connection with and solution by differential equations are explained. Emphasis is then confined to linear equations with symmetric kernel. Solution by Neumann's method is given and the integral equations arising from potential theory are introduced, after which the