Pseudosphärische, Hyperbolisch-Sphärische und Elliptisch-Sphärische Geometrie. By F. Schilling. Leipzig and Berlin, Teubner, 1937. viii+240 pp.

The writer of this review has previously reported on several of Professor Schilling's works for the Bulletin, see for instance vol. 38 (1932), p. 335. The present volume is a further elaboration of the *Pseudo-sphere and non-euclidean geometry*, with the purpose of making this fascinating subject even more attractive to the ordinary trained teacher and student than in the preceding volumes.

In this task Professor Schilling, of the Technische Hochschule of Danzig, has succeeded admirably and I can repeat in praise only what I have said in my previous reviews.

ARNOLD EMCH

L'Emploi des Observations Statistiques. Méthodes d'Estimation. By G. Darmois. (Actualités Scientifiques et Industrielles, No. 356; Statistique Mathématique, exposés publiés sous la direction de Georges Darmois, I.) Paris, Hermann, 1936. 29 pp.

The first part of this booket deals with problems of estimating unknown parameters of frequency distributions from n observations by finding a point in the space of the parameters by the use of a point in the space of the observations, thus giving the best information concerning the position of the "true point."

Part two presents definitions of the distance of an estimation from the true value, and limits of estimations. Comparisons are made of precisions of laws for the mean and median for certain probability laws, together with a comparison of the standard errors of the mean deviation and the standard deviations. The importance of the ratios of two standard deviations is shown.

The next section gives a geometric meaning of the maximum of the function of points in the parametric and observational spaces and shows how to determine the principal part of the standard deviation of an estimation of a parameter which furnishes the maximum of this function. The maximum of this function is compared with R. A. Fisher's maximum of likelihood. The geometric interpretation is very interesting.

The last part defines exhaustive estimation which is equivalent to R. A. Fisher's sufficient estimation. Comparisons are made of two estimations of an unknown parameter by means of the sizes of standard deviations. The quantity $g=E[\partial \log f/\partial m]^2$, plays an important role in the last two sections; f is a frequency function.

W. D. BATEN

Méthode des Caractéristiques pour l'Intégration des Équations aux Dérivées Partielles Linéaires Hyperboliques. By Mlle. Hélène Freda. (Mémorial des Sciences Mathématiques, Fascicule LXXXIV.) Paris, Gauthier-Villars, 1937. 7+82 pp.

The following list of chapter titles gives a good notion of what to expect from this pamphlet: Préface de V. Volterra; Chapitre I. Équations hyperboliques: problèmes aux limites et variétés caractéristiques; Chapitre II. Intégration des équations hyperboliques à deux variables (méthode de Riemann); Chapitre III. Intégration de l'équation

$$\theta(u) = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} - \frac{\partial^2 u}{\partial x_3^2} = f(x_1, x_2, x_3)$$

(méthode de Volterra); Chapitre IV. Quelques extensions de la méthode de Volterra; Chapitre V. Intégration, par la méthode des caractéristiques, de l'équation générale, linéaire, hyperbolique, du type normal; Bibliographie.

The exposition is restricted in general to the solution of second order partial differential equations of the hyperbolic type

$$\sum_{ij} A_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum B_i \frac{\partial u}{\partial x_i} + C - U = f,$$

in which case the *characteristics* used for generating the integral are real. In the general case these characteristic manifolds are *conoids*. When the A_{ij} are constants the characteristic conoid reduces to a cone, and in other more special cases to straight lines.

The pamphlet is the result of a course of lectures given by Mlle. Freda at the University of Rome, 1931. The treatment is classical following the researches of d'Adhémar, Tédoue and Coulon, Picard, and Volterra, with extensions and modifications by Mlle. Freda. Consideration is also given the applications to mechanics and physics.

The exposition is clear but very compact. One unfamiliar with the subject would profitably read a more elementary treatment. As Volterra points out in the preface and as the bibliography indicates, the preparation of this monograph was long and difficult, depending on the researches of many writers spread over a number of years.

Mlle. Freda is to be congratulated not only on her perseverance but also on the excellence of her accomplishment.

V. C. POOR

Elemente der Funktionentheorie. By K. Knopp. Berlin, de Gruyter, 1937. 144 pp.

This little book is Number 1109 in the well-known and highly regarded Sammlung Göschen. The author restricts himself to the treatment of only the simplest parts of the theory that are important for a more detailed and extensive study. The selection of material has been made with the discrimination of a scholar and is written with the clarity of style we have come to expect from the pen of the author.

The thirteen chapters are grouped in five parts: 1. Complex numbers and their geometric representation. 2. Linear functions and the circular transformation. 3. Aggregates and sequences. Power series. 4. Analytic functions and conformal representation. 5. The elementary functions.

To a reader already acquainted with the subject it may seem strange that the integral calculus of complex functions is omitted entirely. However, this enables the author to treat the topics included with amazing thoroughness in the small compass of 135 pages of text. One wishing to go further will find ex-