This suggestion from physics becomes more interesting when we note in pure mathematics that the rigorous formulation of the calculus necessitated assumptions by Dedekind and Cantor which lead to unresolved contradictions. Similar considerations have suggested to this reviewer that the theoretical difficulties at the basis of physics and mathematics may have much more in common than has been realized, and that a clue to their resolution may be found in an alteration in the more general philosophical assumptions common to the two sciences.

F. S. C. NORTHROP

L'Arithmétique dans les Algèbres de Matrices. By Claude Chevalley. (Actualités Scientifiques et Industrielles, No. 323.) Paris, Hermann, 1936. 33 pp.

This book is one of the collection called *Exposés Mathématiques*, published in memory of the late Jacques Herbrand.

This investigation represents another step in the simplification by abstraction of the number theory of linear algebras, a theory which seemed so impossibly complicated when it was first attacked but a few years ago. Ideal theory has grown in importance until, as in the present paper, it constitutes the whole of arithmetic.

Let $\mathfrak{S}$ be a ring with unit element in which every regular element (that is, not a divisor of zero) has an inverse. Then $\mathfrak{S}$ has a regular arithmetic when there is defined a system of modules $\mathfrak{A}, \mathfrak{B}, \cdots, \mathfrak{O}, \cdots$, called ideals such that:

I. The ideals form a groupoid under modular multiplication. The left (right) order of an ideal $\mathfrak{A}$ is the totality $\mathfrak{O}(\mathfrak{O}')$ of elements $\lambda(\lambda')$ such that $\lambda\mathfrak{A}\subset\mathfrak{A}(\mathfrak{A}\lambda'\subset\mathfrak{A})$. The inverse $\mathfrak{A}^{-1}$ of $\mathfrak{A}$ is the set of elements μ such that $\mu\mathfrak{A}\subset\mathfrak{O}'$, $\mathfrak{A}\mu\subset\mathfrak{O}$.

II. If $\mathfrak{A}$ is an ideal, every element of $\mathfrak{S}$ is a product of an element of $\mathfrak{A}$ by the inverse of a regular element of $\mathfrak{A}$.

III. If $\mathfrak{O}$ is a unit of the groupoid, the ideals which have $\mathfrak{O}$ for their left order are the finite left $\mathfrak{O}$-modules which contain regular elements.

IV. In every class of left or right ideals there is an integral ideal prime to any given integral ideal.

The principal result obtained is that if $\mathfrak{S}$ is a total matric algebra over a (not necessarily commutative) field k in which a regular arithmetic is defined, one can define a regular arithmetic in $\mathfrak{S}$. It is known that such a regular arithmetic can be defined in every simple algebra whose centrum is an algebraic field. We see the importance of this result if we recall Wedderburn's theorem that every simple algebra is a total matric algebra over a division algebra.

The above theorem leads to further results in the theory of ideal classes in $\mathfrak{S}$.

C. C. MACDUFFEE

Variationsrechnung und partielle Differentialgleichungen erster Ordnung. By C. Carathéodory. Teubner, Leipzig and Berlin, 1935. 11+407 pp.

A century ago Jacobi, influenced by Hamilton's work on geometrical optics, discovered that there exists a direct connection between the theory of the calculus of variations and the theory of partial differential equations of the first order. Since that time many brilliant mathematicians have contributed to the development of the calculus of variations and some have noticed the connection announced by Jacobi. But it has remained for Professor Carathéodory to give a systematic account of the subject in the book under review.

The book is divided into two parts. The first part (163 pp.) is devoted to partial differential equations of the first order and is an elegant presentation of the theory which is necessary to bring out the relationships with the calculus of variations, which forms the second part of the book. The author's treatment of the simplest problem of the calculus of variations leads directly to the Hamilton-Jacobi equations of mechanics and the foundations of the subject are fully discussed.

The author has not attempted to give a complete treatment of the later developments but has intended to carry them far enough to connect with existing literature. For this purpose the bibliography, containing 190 references to books and articles, is followed by a very helpful section giving advice on the use of the literature.

To quote from the preface: "The purpose which I have pursued in this book will have been accomplished if the student of this field of mathematics may become convinced that there are today three principal aspects of the calculus of variations: first, the calculus of variations of Lagrange, which today forms a part of the tensor calculus; second, the theory of Tonelli, in which the finer aspects of the minimum problem are based on point set theory; then the point of view presented in this book, which is oriented to the theory of differential equations, differential geometry, and physical applications, which was first made prominent by Euler. I hope also to have shown that the Weierstrass theory belongs to this last aspect."

Professor Carathéodory is not only a master of his subject but also a master of presentation. This clearly written fundamental work will prove indispensable for all students of the subject.

W. R. LONGLEY

Introduction to the Theory of Linear Differential Equations. By E. G. C. Poole. London, Oxford University Press, 1936. 200 pp.

This book deals with ordinary linear differential equations and is based on lectures delivered by the author to senior undergraduates at Oxford. Natural prerequisites are an elementary course in differential equations and considerable familiarity with the theory of matrices. The selection of material is such as to form an excellent introduction to this vast and important field of mathematics.

The first half of the book deals with properties common to wide classes of equations. The first chapter, on existence theorems, presents the fundamental theorems for given initial conditions. In the second chapter the solutions of equations with constant coefficients, subject to initial conditions of the Cauchy type, are obtained by the methods of the Heaviside operational calculus. The third chapter is concerned with some formal investigations involving linear operators, adjoint equations, and simultaneous equations with variable coefficients. The next two chapters deal with equations having uniform analytic coefficients, attention being focused on regular singularities. In the second half