

principle an integral over a Cartan locus is varied by slipping the locus over the cylinder, points going along trajectories, while in the Hamilton principle the same expression is integrated along an arc of a trajectory and variation takes place by keeping the end points fixed but slipping the intervening path over the surface of a cylinder, points going over arcs of Cartan loci.

Finally, two points may be noted.

(1) From the form of (23) Whittaker's remarks which were cited follow as a corollary.

(2) Since (23) does not reduce to (16) as a special case, there are for the restricted conditions to which the original paper applies two different characterizing Hamilton extremal integrals.

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NOTE ON THE CANONICAL FORM OF THE
PARAMETRIC EQUATIONS OF A SPACE
CURVE BELONGING TO A NON-
SPECIAL LINEAR LINE
COMPLEX

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In a recent paper,* the author, by means of a projection from hyper-space, obtained the following equations for a general curve belonging to a linear complex,

$$A: \quad x_1 = -t, \quad x_2 = f - \frac{1}{2}tf', \quad x_3 = -f', \quad x_4 = 1.$$

It is the purpose of this note to call attention to a more symmetric form of these equations.

Let $x_i = f_i(s)$ be the equation of a general space curve, and $P_{13} = P_{42}$ be the equation of a general linear complex. If the curve belongs to the complex

$$f_1f_3' - f_3f_1' = f_4f_2' - f_2f_4', \text{ or } \frac{f_1^2(f_1f_3' - f_3f_1')}{f_4^2 \cdot f_1^2} = \frac{(f_4f_2' - f_2f_4')}{f_4^2};$$

* C. R. Wylie, Jr., *Space curves belonging to a non-special linear line complex*, American Journal of Mathematics, vol. 57 (1935), pp. 937-942.

this last can be written in the form

$$\left(\frac{f}{f_4}\right)^2 \cdot u' = v',$$

where $u = f_3/f_1$ and $v = f_2/f_4$. From these we have at once

$$\text{B.1: } f_1 = \left(\frac{v'}{u'}\right)^{1/2} \cdot f_4, \quad f_2 = v \cdot f_4, \quad f_3 = u \left(\frac{v'}{u'}\right)^{1/2} \cdot f_4, \quad f_4 = f_4,$$

$$\text{B.2: } x_1 = (v')^{1/2}, \quad x_2 = v(u')^{1/2}, \quad x_3 = u(v')^{1/2}, \quad x_4 = (u')^{1/2}.$$

To show the equivalence of A and B.2, make the substitutions

$$\text{C.1: } \left(\frac{v'}{u'}\right)^{1/2} = t, \quad \text{C.2: } \frac{df}{dt} = u \left(\frac{v'}{u'}\right)^{1/2} = ut.$$

From C.1 and C.2 we obtain after an easy integration by parts

$$\text{C.3: } v = t \cdot \frac{df}{dt} - 2f.$$

Using these expressions in B.1 we find

$$\text{D: } x_1 = t, \quad x_2 = tf' - 2f, \quad x_3 = f', \quad x_4 = 1,$$

where now the primes indicate differentiation with respect to t instead of s . The slight difference between D and A is due to the fact that in the paper mentioned above, the equation of the complex to which the curve belongs was taken to be $P_{13} + 2P_{42} = 0$.