

## SUPPLEMENTAL REPORT ON ALGEBRAIC GEOMETRY

*Selected Topics in Algebraic Geometry*, II, Bulletin of the National Research Council, Number 96. Supplemental Report of the Committee on Rational Transformations, Virgil Snyder, Chairman. Published by the National Research Council of the National Academy of Sciences, Washington, D. C., November, 1934. 84 pp.

Bulletin number 96 supplements and brings up to date the original report of the Committee on Rational Transformations entitled, *Selected Topics in Algebraic Geometry*, Bulletin of the National Research Council, Number 63.\* That report was so excellent and filled such a pressing need that the Committee was requested by the Council to compile a supplemental report to cover the decade following the period included in the original.

The work of preparing this supplemental report is largely that of the Chairman of the Committee, Virgil Snyder, assisted by Dr. Amos H. Black and Dr. Leaman A. Dye.

The form of this report is similar to that of the original. The discussion is contained in six independent sections, called chapters, of which four are written by Snyder and one each by Black and Dye.

Neither this nor the original report is intended to cover completely the field of algebraic geometry. The title *Selected Topics in Algebraic Geometry* suggests this clearly. Also, as the name of the Committee indicates, the reports are built around the rational transformation. Although the scope of the report is wider than the Committee originally planned, it is still definitely and intentionally limited.

The original report deals with seventeen topics and the supplemental report contains only six. This reduction in the number of topics is due both to omission and concentration.

As stated in the preface, since the recent article, *Algebraische Transformationen und Korrespondenzen*, by Berzolari in the *Encyclopädie der mathematischen Wissenschaften* has so nearly covered the work of the past decade in the transcendental theory and the mapping of algebraic surfaces, these topics are purposely omitted in this supplemental report. The topics omitted are treated in the original report in Chapters 12, 13, 15, 16, 17.

Evidently there was no attempt to relate definitely the material of the remaining twelve chapters of the original to that of the six chapters of the present report. However, this presents no real difficulty, as the subject matter of each chapter is well described in the contents of that report. The supplemental report also treats some subjects not included in the original.

In the supplemental report, the stated contents of the first and sixth chapters indicate an unsystematic arrangement in the treatment of ruled surfaces that does not actually occur. The title of the first chapter should be simply *Curves*, since, in that chapter, ruled surfaces are treated very briefly and only

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\* For a review of Bulletin No. 63, see this Bulletin, vol. 38 (1932), pp. 7-11.

incidentally as related to curves. The discussion of ruled surfaces as such occurs in §2 of Chapter 6.

Also, it might have removed a possible impression of lack of continuity in the arrangement of the material if the subdivisions of the reports had been called *Topics*, as they are in the title, instead of *Chapters*, as they are in the subdivision headings. In most books containing consecutively numbered chapters, each chapter leads to the next. A reader of either report might expect this if he had not read the prefaces, particularly that of the original report in which it is stated, "each chapter is to be regarded as a unit" and "any chapter can be read almost independently of the others."

It should be apparent from the great number of papers cited in these reports that the compilers could not have examined critically all the methods used in deriving the results described therein. With each stated result, there is given the author's name and the reference to the memoir in which the result is obtained. Such citations serve two purposes: first, to enable the reader, to whom a certain result (stated with necessary brevity) is not readily intelligible, to go to the original source where it is discussed in detail; and, second, to indicate that the author cited is credited with this result and is the sole and responsible authority for it.

The good features of the original report are continued in this supplement. Each reference is characterized briefly and clearly and the name of the author given with a superscript referring to the citation. These concise descriptions of the contributions of the respective authors are sufficient, in general, to acquaint the reader with the content of the recent literature of the topics treated. Methods are not given in detail, but are often given in outline.

At the end of each chapter is found a list of the references dealt with in that chapter, each preceded by a number which occurs as a superscript after the author's name in the place where the reference is discussed. A total of 791 books and papers by 454 different authors are cited. The omission of one useful feature that occurred in all but four chapters of the original report, cross references to the section containing the discussion of a paper by a given author, is to be regretted.

There is no index, but the titles of section headings are given in the contents, enabling the reader to find readily the discussion of any given subject. The contents are followed by an alphabetical list of periodicals with abbreviations used in the references. Page xii contains a useful list of books on algebraic geometry published since 1924, arranged alphabetically by authors.

This supplemental report adds greatly to the usefulness and scope of the original. A grand total of 3585 books and papers are discussed and cited in the two bulletins, chiefly on rational transformations and associated topics. These two reports constitute a monumental reference work, by far the most important in the field of algebraic geometry.

Those who prepared this supplemental report, as well as the writers of the original, especially the chairman of the Committee on Rational Transformations, Virgil Snyder, merit the thanks and appreciation of every mathematician. Chairman Snyder and his collaborators have given generously of their time in order that these references in algebraic geometry may be rendered more accessible to workers in that field.

A brief description of the contributions of each author of this supplemental report follows.

VIRGIL SNYDER: CHAPTERS 1, 3, 4, 5

Chapter 1. *Curves and ruled surfaces*, pp. 1–13. The generation, construction, associated correspondences, linear series, and residuation theorems of algebraic curves are treated. Plane curves play a leading role, partly because the problem of the existence of plane curves with assigned characteristics has been seriously attacked in the past few years. Ruled surfaces, generated by trisecants or tangents of one curve or by lines associated with two curves, comprise a comparatively small portion of this chapter.

Chapter 3. *Systems of lines in  $S_n$ ; irregular surfaces*, pp. 25–36. This chapter contains a discussion of generalized complexes, ruled surfaces, and other systems in  $S_n$ , criteria for regularity of surfaces and varieties, involutions on surfaces in  $S_n$ , and the generation of curves, surfaces, and varieties.

The citation 72.1 is a comprehensive treatment of the highly important problem of the irrationality of the cubic primal in  $S_4$  and related problems, itself containing brief characterizations of 112 references by 45 different authors.

Chapter 4. *Cremona transformations*, pp. 37–62. This chapter deals with cremona transformations of the plane and higher space, associated correspondences, reduction and classification of surfaces, basis elements, series of composition in  $S_3$ , invariants of surfaces and varieties, polarities in  $S_4$ , and transformations connected with line complexes in  $S_r$ . Montesano's multiplier is generalized for  $S_n$  and a three-space generalization of the four types of plane involutions is discussed.

Chapter 5. *Multiple correspondences*, pp. 63–72. Involutions of order  $n$  and  $(m, n)$  correspondences, chiefly in the plane and space, are treated, with some extensions to hyperspace. These correspondences are discussed as to methods of generation, irreducibility, and relations to systems of curves and surfaces.

L. A. DYE: CHAPTER 2

Chapter 2. *The mapping of systems of curves*, pp. 14–24. The mapping of systems of curves on a variety was begun by Klein in 1872. By "mapping" is meant establishing a (1, 1) correspondence between the curves of the system and the elements generating the variety. In the discussion, systems are classified according to the order of their component curves. Systems of lines, conics, cubics, quartics, and special curves, chiefly of  $S_3$ , are mapped on varieties of  $S_n$ . These systems of curves include both linear and non-linear systems.

A. H. BLACK: CHAPTER 6

Chapter 6. *Surfaces and varieties*, pp. 73–84. The chapter opens with a discussion of cubic and quartic surfaces and ruled surfaces in  $S_3$ . The finding of definite criteria (as affirmed by Wiman, references 96, 97) determining the completeness of the classification of sextic ruled surfaces is especially important. It is noteworthy that, if Wiman's criteria are valid, all types of sextic ruled

surfaces (scrolls) were obtained thirty years ago by Snyder, who also stated in his final paper,\* "The list of types here found, together with those published in the preceding papers, is believed to be complete."

In the discussion of quintic and sextic (non-ruled) surfaces, the algebraic verification and extension of Montesano's synthetic results for the quintic is of importance.

Under the title *Configurations, surfaces and variations in  $S_n$* , are found six pages of interesting material, difficult to classify. These results largely concern singularities of surfaces and varieties, generation of loci, and generalizations of known properties. The latter part of this section includes some results in differential geometry.

#### ERRATA

- p. v. In title of Chapter I, first word should be "Curves."
- p. 33. First part of line 3 from bottom should read, "to  $n$  given  $S_{n-2}$  in  $S_n$  is a ruled  $V_{n-1}^{n-1}$ ."
- p. 48. In bottom line, superscript should be 235 instead of 158.
- p. 54. In line 7 from bottom, superscript should be 39 instead of 35.
- p. 59. Reference 84 is omitted. This reference is, O. Franceschi, *Il Bollettino di Matematico, Firenze*, (2), vol. 8 (1929), pp. 137-140. See p. 42, line 11 (from bottom) of report.
- p. 60. The author of paper 116 is Hodge (see p. 38, line 7). The name given with 116 belongs with 117.

In addition to the above errata, there are several slight typographical errors especially in superscripts. In general, superscript errors occasion no serious difficulty since author's names accompany respective reference descriptions in the context of each chapter and any name can be readily located in the alphabetical list immediately following that chapter. There is ambiguity, however, when the author has more than one paper in the list. Two cases in which this occurs are included in the errata.

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\* V. Snyder, *On the forms of sextic scrolls having no rectilinear directrix*, *American Journal of Mathematics*, vol. 27 (1905), p. 173.