

the point of view of F. Kaufmann, and then gives an account of the intuitionism of Brouwer and of the results in connection with it of Brouwer, Heyting (the author of the present work), Kolmogoroff, Glivenko, Gödel, Gentzen, de Loor, Belinfante. The second section discusses the classical axiomatic method and the concepts of consistency and categoricity, then proceeds to an account of Hilbert's formal system and the Hilbert concept of a metamathematical proof of consistency. The consistency proofs of Ackermann and von Neumann are outlined; and brief mention is made of the consistency proof of Herbrand; also of the famous theorem of Gödel and its significance in this connection. And the section ends with a discussion of the relationship between formalism and intuitionism. The third section gives a description of several other points of view on the foundations of mathematics, notably those of Manoury and Pasch. The fourth section discusses the relation of mathematics to the natural sciences, comparing the formalistic and the intuitionistic accounts of this relation. At the end of the pamphlet is a five-page bibliography of publications in this field.

ALONZO CHURCH

*The Differential Invariants of Generalized Spaces.* By T. Y. Thomas. Cambridge, University Press, 1934. 241 pp.

This book has a special place in the growing literature on linear displacements in a general manifold of an arbitrary number of dimensions. It deals, as the title indicates, with the differential invariants of such manifolds, thus excluding to a considerable extent special geometrical points of view, and concentrating on the analytical side of the theory. There it throws full light on a field in which the author has distinguished himself for many years through fundamental contributions. This comprehensive account of the present state of things is the more welcome as it is the first of its kind to be written.

The book begins with a general discussion of  $n$ -dimensional spaces as a basis for a theory of differential invariants, starting from a selected set of fundamental postulates. An affine connection is introduced leading immediately into the midst of things: the affine and the projective geometry of paths, Riemannian spaces, spaces with distant parallelism, conformal and Weyl spaces. The immediately following chapters give the foundations of the invariant theories of these spaces, built upon the motion of affine, projective, and conformal relative tensors. Then follow normal coordinates on which the general theory of extension of tensors is based, a theory which leads us from one relative tensor to other tensors by means of differentiation. A next chapter is devoted to an exposition of spatial identities based on the concept of the complete set of identities of the components of an invariant, that is a set of identities furnishing all the algebraic conditions these components satisfy. The simplest example is the complete set of identities of the components  $g_{\alpha\beta}$  of the fundamental tensor of a metric space, which consists of the identity  $g_{\alpha\beta} = g_{\beta\alpha}$ . This leads to complete sets of identities for the curvature tensor of metric space, the projective curvature tensor, and to so-called divergence identities. Then follows a chapter on absolute scalar differential invariants and parameters, which can be considered as defined by means of complete systems of

linear partial differential equations. The theory is given for the case of metric space and space of symmetric affine connection, and requires an introduction into the theory of continuous groups.

This leads up to the two chapters dealing with the equivalence of generalized spaces and reducibility of one type of space to another type. Finite equivalence theorems are given for two affinely connected spaces, for spaces of distant parallelism, and for conformal spaces. Among the theorems on reducibility we mention those on the reduction of an affine space of paths to a metric space and to a Weyl space. The final chapter gives the determination in finite form of the functional arbitrariness of certain sets of quantities consistent with the existence of an affine and a metric space for which they are tensors of a special type. There is also information on the important subject of arithmetic invariants of spaces.

This short account of the contents of the book may perhaps give an impression of the rich variety of subjects, some old, many new, which are discussed by the author. It will, probably for a considerable time, be the standard work of reference on problems relating to the differential invariants of manifolds with a linear connection. But not only those interested in the special research of the author will find in this book material of importance. There is also much worth reading on the theory of partial differential equations, on group theory, and on differential geometry. The explanation of the subjects is quite clear, and the use of small print for those passages which only supplement the main ideas of the text emphasizes the efforts of the author to minimize the difficulties that will confront the reader who wishes to acquaint himself with the chain of ideas exposed in the text.

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