

facts presented are already well known through articles and books in English. These studies, however, have been woven together into a whole at the same time that the results of more recent investigations have increased their value. A study of language and script has been added. The chapter headings cover: *Babylonische Rechentechnik*; *Allgemeine Geschichte Sprache und Schrift*; *Zahlensysteme*; *Ägyptische Mathematik*; *Babylonische Mathematik*.

This work has broadened the world for the historian of mathematics. For this, as for much more, the historian is greatly indebted to the author.

LAO G. SIMONS

Aufgabensammlung zur höheren Algebra. By Helmut Hasse. Sammlung Göschel, vol. 1082. Berlin, de Gruyter, 1934. 175 pp.

The present collection of problems is a sequel to the author's two-volume treatise on algebra in the Sammlung Göschel. The main headings are: *Theory of fields and rings*, *Groups*, *Linear algebra and determinants*, *Roots of algebraic equations including the solution of equations by radicals*. Throughout the author takes the point of view of abstract algebra, trying to make the problems supplement and extend the theories of his textbook. Some problems are strikingly new. It is for instance rather surprising to find given as problems Zassenhaus' proof for the generalized Jordan-Hölder theorem and Witt's proof for Wedderburn's theorem that any field with a finite number of elements must be commutative.

Some problems are quite simple, but on the whole they become increasingly difficult until one reaches the last stage at the final problem where the author confesses that he cannot solve it himself. The book may be recommended to anyone interested in abstract algebra and it should form a suitable supplement to a more advanced course in higher algebra.

OYSTEIN ORE

Mathematische Grundlagenforschung. Intuitionismus. Beweistheorie. By A. Heyting. Berlin, Springer, 1934. iv+73 pp.

This pamphlet is one of the series *Ergebnisse der Mathematik* published by the editors of the Zentralblatt. It deals chiefly with the foundations of mathematics, and mathematical logic, from two points of view, the intuitionism of Brouwer and the formalism of Hilbert, and gives an able, clear, and concise account of the essentials of these two points of view and of the important results which have been obtained in connection with them. As explained in the introduction, no attempt is made to give an account of the logistic formulation of the foundations of mathematics, a subject which is to be treated in a later number of the series.

This work is recommended, not only to mathematical logicians, but also to mathematicians in general who desire an understandable survey of its field. The reviewer knows of no better such survey, indeed of none nearly so good.

The first section begins with a notice of Poincaré as historical forerunner of intuitionism, describes the point of view of the French semi-intuitionists as they are here called (Borel, Lebesgue, Baire), the first theory of Weyl, and

the point of view of F. Kaufmann, and then gives an account of the intuitionism of Brouwer and of the results in connection with it of Brouwer, Heyting (the author of the present work), Kolmogoroff, Glivenko, Gödel, Gentzen, de Loor, Belinfante. The second section discusses the classical axiomatic method and the concepts of consistency and categoricity, then proceeds to an account of Hilbert's formal system and the Hilbert concept of a metamathematical proof of consistency. The consistency proofs of Ackermann and von Neumann are outlined; and brief mention is made of the consistency proof of Herbrand; also of the famous theorem of Gödel and its significance in this connection. And the section ends with a discussion of the relationship between formalism and intuitionism. The third section gives a description of several other points of view on the foundations of mathematics, notably those of Manoury and Pasch. The fourth section discusses the relation of mathematics to the natural sciences, comparing the formalistic and the intuitionistic accounts of this relation. At the end of the pamphlet is a five-page bibliography of publications in this field.

ALONZO CHURCH

The Differential Invariants of Generalized Spaces. By T. Y. Thomas. Cambridge, University Press, 1934. 241 pp.

This book has a special place in the growing literature on linear displacements in a general manifold of an arbitrary number of dimensions. It deals, as the title indicates, with the differential invariants of such manifolds, thus excluding to a considerable extent special geometrical points of view, and concentrating on the analytical side of the theory. There it throws full light on a field in which the author has distinguished himself for many years through fundamental contributions. This comprehensive account of the present state of things is the more welcome as it is the first of its kind to be written.

The book begins with a general discussion of n -dimensional spaces as a basis for a theory of differential invariants, starting from a selected set of fundamental postulates. An affine connection is introduced leading immediately into the midst of things: the affine and the projective geometry of paths, Riemannian spaces, spaces with distant parallelism, conformal and Weyl spaces. The immediately following chapters give the foundations of the invariant theories of these spaces, built upon the motion of affine, projective, and conformal relative tensors. Then follow normal coordinates on which the general theory of extension of tensors is based, a theory which leads us from one relative tensor to other tensors by means of differentiation. A next chapter is devoted to an exposition of spatial identities based on the concept of the complete set of identities of the components of an invariant, that is a set of identities furnishing all the algebraic conditions these components satisfy. The simplest example is the complete set of identities of the components $g_{\alpha\beta}$ of the fundamental tensor of a metric space, which consists of the identity $g_{\alpha\beta} = g_{\beta\alpha}$. This leads to complete sets of identities for the curvature tensor of metric space, the projective curvature tensor, and to so-called divergence identities. Then follows a chapter on absolute scalar differential invariants and parameters, which can be considered as defined by means of complete systems of