*Leçons sur les Progrès Récents de la Théorie des Séries de Dirichlet.* By Vladimir Bernstein. Avec une préface de Jacques Hadamard. (Collection de Monogrammes sur la Théorie des Fonctions). Paris, Gauthier-Villars, 1933. xiv +320 pp.

During the last two decades, many interesting papers have appeared dealing with the relationship between a Dirichlet series and the singularities of the analytic function of a complex variable it represents. The book under review gives for the first time a systematic account of all the results obtained by various authors, allowing the reader to get acquainted quickly with a field where many important questions are as yet unanswered. Following the tradition of the "Collection Borel" in which the book appeared, it is not written exclusively for specialists but also for those mathematicians who wish to approach the subject matter for the first time. All necessary auxiliary methods and theorems are completely discussed, some of them in three appendices avoiding the interruption of the exposition of the theory proper. Very clearly written and complete, this book will be of interest to all mathematicians.

In the first chapter a brief account is given of the classical elementary properties of Dirichlet series. In Chapter 2 the author analyses the structure of the sequence of the exponents (maximal density, index of condensation) and, in connection with the overconvergence of Dirichlet series, its influence on the singularities of the function. Chapter 3 deals with the integral representation of the function $\sum a_n\phi(\lambda_n) \exp(-\lambda_n z)$ in terms of $\phi(z)$ and $\sum a_n \exp(-\lambda_n z)$, that is, with the theorem of Cramer and all its generalizations. Chapter 4, beginning with Landau's theorem on Dirichlet series with positive coefficients, deals with the influence of the frequency of variations of signs among the coefficients. Chapters 5 and 6 are devoted mainly to the work of the author on the relative position of the line of convergence and the line of holomorphy, as well as the frequency of singularities on the latter for series whose exponents have a finite maximal density. Some of the results of these chapters are new and new proofs are given for some known theorems. The reviewer has, however, been unable to follow the proof for the necessity of the existence of the function $\phi(z)$ mentioned in the fundamental theorem on pages 103 and 104. The results obtained in the course of the proof are sufficient, however, to guarantee the truth of all the different applications of this theorem that are made in the sequel. In Chapter 7 the restriction of finite maximal density is dropped and the more or less isolated results which are known for the most general case are taken up. Chapter 8 deals with the composition of singularities, that is, generalizes the classical theorem of Hadamard for power series. With the last chapter we leave the theory proper and consider applications to the general theory of analytical functions, in particular to functions of the exponential type. Some of the theorems obtained are the more interesting since no direct proof for them is known.

The three "notes" at the end of the book are devoted respectively to a complete study of sequences of finite maximal density, to the general properties of the function $\Pi(1-z^2/\lambda_n^2)$, which plays such a fundamental role in the whole theory, and finally to a short account of the transformation of Laplace.

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