SHORTER NOTICES

Lezioni di Analisi. By Francesco Severi. Volume I. Bologna, Zanichelli, 1933. viii+434 pp.

Just what one may expect to find in a volume which bears the title *Lezioni di Analisi*, seems to be in general an uncertain quantity. It depends upon the whims and interests of the individual writing the volume. About all that seems to be guaranteed is that at some point or other, one will be introduced to some of the fundamental facts concerning real function theory, that is, questions of limits and continuity, derivatives, and possibly integrals. The volume under review is no exception.

A casual glance through the headings of the chapters is perhaps illuminating. They read in succession: *Combinatory Analysis, Determinants and Linear Equations, Real Numbers, Complex Numbers, Functions and Limits, Derivatives and Differentials of Functions of one Variable, Numerical and Taylor's Series, Preliminary Notions on Integrals, Algebraic Functions*. Considering the fact that the last chapter covers over a hundred pages, we have here a volume in which more than a third of the space is devoted to matters of algebra, and the remaining two thirds to functional analysis.

The method of presentation adopted is to give the simple and pertinent facts concerning the topics under consideration in the main body of the chapter, and to relegate to the last paragraph, headed supplement and exercises, additional ideas which the author considers worthy of mention. The result is that frequently very elementary facts rub elbows with profound and abstract considerations. For instance, the chapter on functions and limits gives a thorough presentation of the fundamental facts of function theory and in its supplement takes up such things as the Bolzano-Weierstrass Theorem and the Borel Theorem in higher dimensions, discusses the Zermelo Axiom of Selection, and finishes up with allusions to notions of topology and analysis situs. On the other hand, the succeeding chapter on derivatives is of a most elementary nature, while the following chapter on integration is limited to the simpler methods of evaluating indefinite integrals. Apparently this brief chapter on integrals is intended to salve the author's conscience on having fulfilled some unwritten requirement of including integrals in this volume, and permit him to take up topics close to his heart at greater length, namely, the basic matters pertaining to algebraic functions as a foundation for algebraic geometry.

The last chapter, devoted to algebraic consideration of algebraic functions, is one of the distinguishing features of the volume. It takes up in detail matters of divisibility of polynomials in one and many variables, matters of elimination and resultants from different points of view, discusses the continuity of the roots of an algebraic equation as a function of the coefficients, gives a simple proof of the fundamental theorem of algebra and the general solution of equation of the third and fourth degree, and winds up with the approximation of roots of algebraic equations. The extensive supplement to this chapter includes geometric interpretations of some of the processes involved in the chapter itself. It is an excellent treatment of the subject under consideration.

The volume as a whole suffers from a lack of unity, even greater than that usually associated with treatises of this kind. This is due to the desire of the author to cater to the elementary as well as to the advanced student, and to his prejudice in favor of algebraic analysis. On the other hand, the author has managed to include and touch upon many different mathematical notions and has laid out before the prospective student of mathematics an adequate array of interesting directions for further edification and investigation, which, after all, might be considered a justification of a set of Lessons in Analysis.

T. H. HILDEBRANDT

Die Grundlagen der Geometrie, ihre Bedeutung für Philosophie, Mathematik, Physik und Technik. By Hugo Dingler. Stuttgart, Ferdinand Enke, 1933. viii+76 pp.

The author proposes a treatment of the foundations of geometry of a radically new kind, starting, not from a set of axioms whose consequences are to be derived, but from a description and analysis of the experimental process of measuring extended bodies. On this basis he proves that the geometry of physical space is euclidean. "Alle die vielen von Einstein und seiner Schule ohne Überlegung und ohne Erröten vorgebrachten Behauptungen über die Geometrie sind damit für jeden, der noch konsequent und umfassend denken kann, inhaltslos und unmöglich geworden."

The principal error seems to lie in the definition of the concept of a solid body. A body is to be called a solid body if it has the property that, if it is translated along any straight line g and a plane through g, then it is translated at the same time along every plane which passes through g and along every parallel to g which lies in one of these planes through g and in or on the body. Assuming the existence of bodies which are solid in this sense, the author is able to prove the euclidean parallel postulate!

ALONZO CHURCH

Algebraic Functions. By Gilbert Ames Bliss. American Mathematical Society Colloquium Publications, Volume 16. New York, 1933. vi+218 pp.

This book, immediately striking for its conciseness, is one of the most remarkable works ever produced on the subject of algebraic functions and their integrals.

The distinguishing feature of the book is its third chapter, on rational functions, which gives an extremely brief and clear account of the theory of divisors. Here the integrands of the three elementary types of abelian integrals are set up by the arithmetic methods of Dedekind and Weber. The arithmetic treatment is definitely simpler and more elegant than the potential-theoretic method of Riemann, or the geometric method of Brill and Noether which is based on the reduction of the singularities of algebraic curves. The theory of divisors, hitherto available chiefly in the ponderous classic treatise of Hensel and Landsberg, is presented by Bliss in hardly more than thirty pages.

A very readable account is given of the topology of Riemann surfaces and of the general properties of abelian integrals. Abel's theorem is presented, with