HOPFNER ON GEODESY

Physikalische Geodäsie. By F. Hopfner. Mathematik in Monographien und Lehrbüchern. No. 15. Leipzig, Akademische Verlagsgesellschaft, 1933. ix+434 pp.

The type of textbook on geodesy most familiar to students in this country is a sort of practical manual of large-scale higher surveying. Often considerable attention is given to the details of observation. The point of view of such books is predominantly geometric; naturally there is little space for extended theoretical developments, often the proofs of formulas have to be omitted.

More theoretical is the type of book best exemplified by the classical treatises of Helmert and Clarke. In these there is more of theory and a nearer approach to mathematical rigor, also less attention to observational detail, but the points of view of the engineer in the field and of the computer or the theorist are not entirely forgotten. This comprehensiveness of outlook is one reason why these two works retain their value fifty years and more after their publication.

Hopfner's book takes us further over into the theoretical side than either Clarke or Helmert, or indeed than any book known to the reviewer calling itself a treatise on geodesy. This is said by way of information, not of blame; the author is entitled to choose his own treatment and the first eight chapters are well done within the limitations of the space available, though the question might arise as to how much of the material of these chapters is really geodesy, or to what extent it prepares the way for a study of geodesy.

Chapter I treats of the relation of line and surface integrals, Green's theorem, the Laplacian operator, and elliptic coordinates. Chapter II deals with spherical harmonics; Chapter III with Lamé's functions; and Chapter IV with attraction and potential, and also with Green's functions and Dirichlet's principle. Chapter V, a brief one, deals with the attraction and potential of a homogeneous ellipsoid. In Chapters VI, VII, and VIII, uniform rotation is introduced, and we have a discussion of the figures of equilibrium of rotating homogeneous liquids: Maclaurin's ellipsoid, Jacobi's ellipsoid with three unequal axes, Poincaré's pear-shaped figure, etc. The foot-notes give welcome references to the more recent literature. All this is interesting, but how much connection with geodesy it has is not so clear; the author does not discuss the possible cosmogonic implications, which are usually considered to constitute the most interesting applications of these theoretical developments.

Chapter IX treats Clairaut's problem, the figure of equilibrium and the form of the level surfaces for a slowly rotating mass of self-attracting liquid. The outer surface and the level surfaces within are approximately ellipsoids of revolution. Clairaut's differential equation connecting the ellipticity with the equatorial radius of any given level surface is established and integrated for several well known laws of density that make the results mathematically tractable. These laws suppose continuous variation in density from center to free surface. Seismologists look with more favor on discontinuities in density, but this physically interesting case is passed over, except for references to the literature. Radau's ingenious transformation of Clairaut's equation is given;

it shows why very different laws of density give so nearly the same result, as far as the external gravitational field is concerned, that the observed character of the latter is no test of the merit of one or another law.

These nine chapters occupy 287 of the 429 pages in the book (exclusive of preface, table of contents, and index.) So far the reviewer has found nothing to which he could reasonably object and much to commend, but the two remaining chapters are full of controversial matter. Even the reader approaching the subject for the first time might suspect something of the sort, but he would find little help in the book if he wished to learn the other side.

Chapter X contains an error that will be more readily appreciated by mathematicians than will the fine points of the controversy about the more technically geodetic matters in Chapter XI. In Section 1 of Chapter X the author "proves" the existence of a Newtonian potential function expressed as a series of spherical harmonics that is supposed to represent within a certain region the potential both inside and outside of attracting matter. The method of obtaining this series is very simple in certain cases, especially those where the matter is homogeneous. Unfortunately it will not work, as would be shown by a comparison with the known and correct result for the external potential of a homogeneous ellipsoid obtained by a different method and given by the author himself in Section 5 of the same chapter.

This error vitiates much of Chapter XI, which follows and which is by far the longest chapter in the book. The Chapter is entitled "Das Problem von Bruns." Bruns published in 1878 a very useful little work of some fifty pages called *Die Figur der Erde*; in it he emphasized the physical rather than the purely geometric aspect of the problem and reminded geodesists that their problem involved more than the form of one level surface of the earth's field of force, in short that it included the entire field of force. Bruns's work was valuable although he made one grave error of potential theory, an error that Hopfner unfortunately adopted as his own and defended in his articles published in periodicals. Hopfner has repeatedly called attention to the value of Bruns's work. But, in spite of Bruns's service to geodesy, it seems to the reviewer that it is hardly justifiable to call a certain equation for the warping of the geoid "Bruns's theorem" and a correction based on this equation "Bruns's term," as Hopfner does. Stokes in his classic article *On the variation of gravity at the surface of the earth*, (Transactions of Cambridge Philosophical Society, vol. 8, p. 72, or Mathematical and Physical Papers, vol. 2, p. 131), nearly thirty years earlier than Bruns, used the same equation and the same term, and he did not even pause in the course of his exposition to call special attention to them.

More important, however, to the reader using this book as an introduction to the subject, is the fact that the author bases much of this chapter on the erroneous theorem already mentioned and makes other statements not generally accepted. He refers to some of his numerous articles along the same lines, but does not indicate that his conclusions have been widely questioned. It would take far too much space even to summarize the controversy. All that the reviewer can do is to put the reader on his guard, and to refer him to the volumes of Gerland's *Beiträge zur Geophysik* as far back as 1929; there he will find numerous articles by Hopfner and by those who take issue with him.

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