Oeuvres de Georges Humbert publiées par les soins de Pierre Humbert et de Gaston Julia. Tome I avec un Préface de Paul Painlevé. Paris, Gauthier-Vlliars, 1929. ix+555 pp.

The eminent French mathematician Georges Humbert who died early in 1921 was the discoverer of a host of elegant and profound geometric and arithmetic facts. Before him the domain of application of algebraic integrals (in particular by means of Abel's theorem), of the theta functions of Poincaré, and of singular Abelian functions had not been exploited in their rich variety of detail. It was here that Humbert was remarkably successful. As two instances of the geometric results contained in the volume under review which are of such extraordinary simplicity as to involve only the rudiments of the calculus, I may mention the following.

The plane algebraic curves whose arc length is a rational function of the coordinates are precisely the caustics by reflection of plane algebraic curves for parallel incident rays.

If a paraboloid of revolution intersects a sphere of radius r in two closed curves, the difference of the spherical areas cut out is independent of the relative orientation of the sphere and paraboloid, and is in fact $4\pi r p$, where p is the parameter of a meridian parabola of the paraboloid.

This volume contains Humbert's work concerning algebraic curves and Abel's theorem. The later volumes are to contain his work on Abelian functions and their applications, and his work in the theory of numbers. To the theory of numbers Humbert devoted the last twenty years of his life, and more than one third of his 45 listed papers fall in this field.

As Painlevé says in his *Préface*, "As long as men live capable of cultivating mathematics, they will enjoy and admire the perfection of such a work!"

G. D. Birkhoff

Asymptotische Gesetze der Wahrscheinlichkeitsrechnung. By A. Khintchine. Berlin, Julius Springer, 1933. 77 pp.

The book opens with the Laplace-Liapounoff limit theorem on the approach to the normal probability function of the distribution of the sum $x=x_1+x_2+\cdots+x_n$ of n variates (n large) that are quite independent of the special properties of the distribution functions $F_1(x_1)$, $F_2(x_2)$, $F_n(x_n)$ of the variables $x_1, x_2, \cdots, x_n$, but are subject to certain conditions concerned with the weight and expected value of any one variate in relation to their sum. It is held that the limit theorems known as the asymptotic laws of probability are not an incidental part of the subject, but, on the contrary, that they form an essential part of the science.

The book is devoted fundamentally to the unification of the subject of asymptotic probability. For this purpose use is made of the fact that the probability function

$$\phi\left(\frac{x}{z^{1/2}}\right)=\frac{1}{2\pi}\int_0^{x/z^{1/2}} e^{-u^2/2}du, \qquad (z>0),$$

satisfies the differential equation

$$\frac{\partial \phi}{\partial z} = \frac{1}{2} \frac{\partial^2 \phi}{\partial x^2}.$$

In the various questions of analysis that arise, the author has used the notion of upper and lower functions whose significance for theoretical probability was recently discovered by Petrowsky.

Continuous stochastic processes are shown to lead to distributions given by the normal probability function while discontinuous stochastic processes are shown to lead to distributions given by the Poisson exponential function. Much of Chapter IV deals with the distribution of chance fluctuation restricted as to direction, and with a generalization of the LaPlace-Tchebycheff proposition concerning the approach of the probability function for the sum of n variables $x = x_1, x_2, \cdots, x_n$ to a function v which satisfies the differential equation

$$\frac{\partial v}{\partial t} + \frac{1}{2} \frac{\partial^2 v}{\partial x^2} = 0.$$

The fifth chapter gives a proof of the theorem giving the probability of an upper bound of $|s_m|$, where $s_n = x_1 + x_2 + \cdots + x_n$, in terms of repeated logarithms.

The theorems developed relate in many cases to the probability of events occurring in relation to an assigned time, and are thus connected with the diffusion problems to which the whole of Chapter III is devoted.

H. L. RIETZ

Questions non résolues de Géométrie Algébrique. By L. Godeaux. Actualités scientifiques et industrielles, No. 77. Paris, Hermann, 1933. 24 pp.

This little pamphlet contains an accurate and well written account of the present state of three famous unsolved problems:

Involutions in space.

Conditions for rationality of a three-dimensional variety.

Demonstration of the irrationality of the general cubic variety in four-way space.

The bibliography contains titles of 12 books and of 71 recent articles.

VIRGIL SNYDER

Vorlesungen über Algebra. Unter Benutzung der dritten Auflage des gleichnamigen Werkes von Gustav Bauer in fünfter vermehrter Auflage dargestellt von L. Bieberbach. Leipzig, Teubner, 1933. x+358 pp.

The fifth edition of Bauer's *Algebra* differs very little from the fourth.* The principal change has been the rewriting of Chapter 2 of Section 1 to include a discussion of number fields and rings. On the basis of this addition the author has made minor changes throughout and has included in Chapter 5 of Section 1 a second proof of the fundamental theorem of algebra. There are also a few trivial changes in the chapters on linear equations, matrices, and groups, and a number of additional references to original articles.

L. T. MOORE

* Reviewed in this Bulletin, vol. 35 (1929), p. 581, by T. H. Gronwall.