The *Catalogue* of the exhibition in London is much more attractive in every way. The notes for the 691 entries are more profuse and the additional material is of special interest. There were quite a number of items neither manuscript nor printed.

The verse of Dodgson has been collected in a single volume with an introduction by J. F. McDermott (New York, 1929). Much of this verse has been set to music,* some of which is to be found on phonographic records. The number of mathematicians who have written verse, or dramas, is quite large. But the number of those who have produced a great effect on their country's literature, like Lewis Carroll in the writing of nonsense, is exceedingly small.

R. C. ARCHIBALD

PICARD ON CURVES AND SURFACES

Quelques Applications Analytiques de la Théorie des Courbes et des Surfaces Algébriques. By M. Emile Picard. Paris, Gauthier-Villars, 1931. viii +224 pp.

This volume is published as Fascicule 9 of the *Cahiers scientifiques.* It contains the course given at the Sorbonne in 1930. The first chapters discuss Abelian integrals and the problem of inversion for $p=1$ and $p>1$. After mentioning the theorem of Jacobi that a single-valued function in one variable can have no more than two distinct periods, the corresponding theorem for two variables is proved. Its statement is that a single-valued function of two variables can have no more than four pairs of distinct periods. Furthermore, it is proved that if one imposes upon a quadruply periodic function the condition of being rational at any finite point, there necessarily exists a relationship bebetween the four pairs of periods. A set of equations expressing this relationship is found.

Chapter 3 mentions the known theorems (a) that the coordinates of a point of an algebraic curve of genus zero are expressible as rational functions of a single parameter, (b) that the coordinates of a point of a curve of genus one are expressible by doubly periodic functions which are meromorphic at any finite point. In these two cases, the functions which give the parametric representation have only isolated poles or singular points throughout the whole plane. Then follows the more general theorem due to Poincaré, (c) that the coordi nates of any point of an algebraic curve can be expressed by single-valued functions of a single parameter, but when the genus is greater than one, the essential singularities of these functions are no longer isolated.

The solution of the differential equation

$$F\left(u, \frac{du}{dz}\right) = 0,$$

which possesses a meromorphic integral at any finite point of the plane, requires nothing but exponential functions and doubly periodic functions. The solution of

* The work of Williams and Madan lists (pp. 263–281) nearly 70 "dramatizations and musical settings" of Carroll's *Alice* and verse.

$$F\left(u, \frac{d^2u}{dz^2}\right) = 0,$$

integrable by meromorphic functions, requires nothing but the classical transcendental functions.

Chapter 4 demonstrates some theorems on harmonic functions and considers the equation $\Delta u = ke^u$.

The next chapter begins by stating the theorem: To any algebraic surface, $f(x, y, z) = 0$, there always corresponds by means of a birational transformation a surface $F(X, Y, Z) = 0$ having no singularities but a double curve which may contain a certain number of triple points. It is allowable, of course, that these singularities are the most general ones of their kind, that is, the tangents to the double curve at a triple point are distinct and not situated in the same plane. Moreover, the tangent planes at a point of the double curve are distinct except, perhaps, for a limited number of points called *pinch points*. Let us examine some non-decomposable surfaces of the fourth order whose plane sections are unicursal curves. These curves being permitted to have three double points, there must exist a double curve of the third order which can not be a plane curve, for then any straight line of the plane would cut the surface in six points. If this cubic is not degenerate the surface is ruled, for through any point M of the surface there passes a double secant of the cubic, and since this line then meets the surface in 5 points it lies entirely in the surface. If the cubic degenerates into a straight line and a conic, the straight line necessarily meets the conic, otherwise, through the trace of the line on the plane of the conic, an infinity of straight lines could be drawn cutting the surface in 6 points. We see as before that the surface is ruled. It is easy to form the equation of surfaces of this kind in these two cases. Let $f_1 = 0$, $f_2 = 0$, and $f_3 = 0$ be the equations of three quadrics passing through the cubic but not belonging to the same pencil. Any equation of the form

$$Af_1^2 + Bf_2^2 + Cf_3^2 + Df_1f_2 + Ef_2f_3 + Ff_3f_1 = 0, \tag{1}$$

where $A, B, \cdots, F$ are constants, represents a surface of the fourth order possessing the cubic as a double curve. Finally let the cubic degenerate into three straight lines. These lines must be concurrent and not coplanar. It may be shown that there can be no other lines on the surface but those passing through the triple point. Cones which have a plane unicursal quartic for a base are surfaces of the kind represented by equation (1). Besides there exist non-ruled surfaces of the fourth order which possess three double lines. By taking these lines as axes of coordinates, equation

$$Ax^2y^2 + By^2z^2 + Cz^2y^2 + xyz = 0$$

represents such a surface, which is not a cone, and which can not contain any straight lines but the double lines. These problems are then generalized.

The theory of algebraic curves leads to the study of Abelian integrals. The theory of surfaces leads to an analogous study of the total differential equation $P(x, y, z)dx + Q(x, y, z)dy = 0$, where P and Q are rational functions of x, y, and z, and where z is supposed to be replaced by its value taken from the equation of the surface $f(x, y, z) = 0$. The functions P and Q also satisfy the condition of integrability $\partial P/\partial y = \partial Q/\partial x$. The study of these expressions leads to results which have no equivalent in the study of curves.

Integrals of the first kind are defined by the condition that they keep a finite value at every finite or infinite point of the surface. Picard then finds several conditions that must be met in order that there may exist integrals of the first kind. Examples are given of surfaces which do meet these conditions.

Integrals of the second kind for a total differential are those whose value along a path, which is reducible to a point by continuous deformation, is zero. Such integrals exist and may be found but a lengthy discussion shows that *in general* a surface has no integrals of the second kind. Any integral of the rational total differential $Rdx+Sdy$ which does not meet the conditions of the first two kinds is called an integral of the third kind.

One can draw on a surface, with ordinary singularities, particular curves $C_1, C_2, \cdots, C_\rho$, such that there exists no integral of the third kind for the total differential having as specific logarithmic curves the totality of curves C or a part of them but such that there does exist an integral of the third kind having for specific logarithmic curves a $(\rho+1)$th arbitrary curve Γ and the totality of curves C or a part of them.

The two final chapters discuss the double integrals of rational functions $\int\int F(x, y)dx\,dy$ and $\int\int R(x, y, z)dx\,dy$, where $f(x, y, z)=0$.

The book ends with three notes which had previously appeared in mathematical journals.

F. A. FORAKER

FOWLER ON STATISTICAL MECHANICS

Statistical Mechanics. By R. H. Fowler. Cambridge University Press, 1929. 570 pp.

The Adams Prize in the University of Cambridge for 1923–1924 was awarded to Mr. R. H. Fowler for an essay dealing with the properties of matter at high temperatures. The essay was subsequently developed into the extensive systematic treatise before us for review. The author, with the occasional collaboration of other scientists generously acknowledged in his preface, has prepared a detailed survey of a very large portion of the existing theoretical and experimental material concerning the behavior of matter in bulk. The mechanical principles on which the treatment is based are those of the classical and the Bohr-Sommerfeld theories; the essential modifications necessitated by the newer quantum theory are discussed in the final chapter. It should not be supposed, however, that this point of view detracts seriously from the fundamental value of the book; for, essentially the same statistical methods are effective in the new quantum theory as in the others and the results obtained upon the introduction of the Bose-Einstein and Fermi-Dirac statistical weightings are often only slightly different from those obtained in the older quantum theory. If the author should set out to revise his treatment of those instances where the principles of the present quantum theory produce essential changes, he would run today the same risk that he incurred in 1926–1929 of seeing the basic physical principles of his work supplanted almost before the last pages of the manuscript reached the printer.