

Interpolation und genäherte Quadratur. By Dr. Gerhard Kowalewski. Leipzig, Teubner, 1932. 146 pp.

This book is a distinct addition to the theory of interpolation and approximate quadrature. The treatment is what we should expect of the author—a scholarly treatment of the theoretical side of the subjects. He lays particular stress upon the exact representation of the error term for which most books give only an estimate. The author stresses the importance of the Euler-Maclaurin formula, for which he gives a new derivation. This formula is often treated in a superficial manner in works on the calculus.

The subject is treated entirely by the calculus of differences and the results are given in a new form. The subjects treated are the following: 1. Interpolation by Polynomials. 2. Newton's Quadrature Formulas. 3. Euler-Maclaurin's and Boole's Theorems. 4. The Euler-Maclaurin's and Boole's Theorems as source of Quadrature Formulas.

The treatment is purely theoretical and will be of great value to persons investigating the subject. The author thinks the book will be of service to all who are concerned with applied mathematics, such as physicists, chemists, engineers, biologists, psychologists, and physiologists. The writer of this review is inclined to think that this will not be the case since few persons except mathematicians will be able to follow the rather abstract treatment of the author. Then, too, there is a dearth of examples (only three in the whole book), which is a great disadvantage to persons who are studying the subject more for its applications than for the theory.

In the second chapter the author considers what he terms "Newton's Favorite Formula," more commonly called "Simpson's Three-Eighths Rule" in the English books. Apparently he considers this formula to give more accurate results than Simpson's One-Third Rule—called by the author merely "Simpson's Rule." To show the comparative accuracy of the two rules he works out an example, $\int_0^\pi \sin x \, dx = 2$, by Simpson's Rule (with three ordinates), and by Newton's Favorite Formula (with four ordinates) and obtains the values $2\pi/3 = 2.09$ by the first formula and $3\pi\sqrt{3}/8 = 2.04$ by the second. However, it is evident that this is not a fair comparison. If we take seven ordinates for each formula, we should get by Simpson's Rule $\pi(4 + \sqrt{3})/9 = 2.00086$ and by Newton's Favorite Formula $\pi(5 + 3\sqrt{3})/16 = 2.00201$, so that the error by the former rule is less than one-half that by the second.

The greater exactness of Simpson's Rule over the Three-Eighths Rule has been previously noted. See, for example my review of Scarborough's *Numerical Mathematical Analysis* (American Mathematical Monthly, vol. 38 (1931), pp. 396–402).

E. B. ESCOTT

Origine des Rayons Gamma. Structure Fine du Spectre Magnétique des Rayons Alpha. By Salomon Rosenblum. (Exposés de Physique Théorique, No. 4.) Paris, Hermann, 1932. 37 pp.

The memoir is one of a series of theoretical physical presentations by various authors and is edited by Louis de Broglie of the Sorbonne, a Nobel Laureate.

The author presents historically the subject of the study of magnetic spectra of alpha particles. In particular, he describes his own excellent researches with the large Academy of Science magnet on the fine structure of the magnetic spectra of alpha particles. He shows the correlations between the differences of energies of the various groups of alpha particles of several radioactive bodies and the γ -rays from these bodies which have otherwise been studied, and points out the agreement with Gamow's theory. Gamow predicted in his theory of the disintegration of a radioactive atom that γ -rays should be emitted in the case of those atoms for which the fine structure in the ranges exists, presupposing different energy levels in the nucleus. The emission of the α -particles from these levels accounts for the fine structure of the magnetic spectra of alpha rays. A fall of an α -particle to a lower level in the nucleus characterizes the quantum emitted as a gamma ray.

A complete bibliography is given.

A. F. KOVARIK

Cartesian Tensors. By Harold Jeffreys. Cambridge, The University Press, and New York, Macmillan, 1931. vi + 92 pp.

This little book is devoted mostly to applied mathematics. The subject content is indicated by the chapter headings which are as follows: Cartesian Tensors, Geometrical Applications, Particle Dynamics, Dynamics of Rigid Bodies, Equivalence of Systems of Forces, Continuous Systems, Isotropic Tensors, Elasticity, and Hydrodynamics.

The discussion throughout is confined to a euclidean three-space and with reference to rectangular cartesian coordinate systems. Hence the tensor quantities are with reference to the group of orthogonal transformations. According to the preface, "the object of this work is to illustrate the use of tensor methods." In the opinion of the reviewer it is scarcely possible to accomplish this object when no distinction is made between covariant and contravariant tensors. The significance of tensors is necessarily missed unless the discussion is with respect to transformations of coordinates at least as general as the group of affine transformations. In slightly more space, the author could have given an introduction to tensor analysis useful to a beginner. Certain elementary but fundamental things such as a basis of a linear vector space, and the usual interpretations of the coefficients of contravariant and covariant vectors in terms of the base vectors and the reciprocal system, are most clearly presented by considering a euclidean three-space and the group of affine transformations. Then by specializing the transformations to the orthogonal group the formal results concerning tensors which the author gives would result, and at the same time the beginning student would see why the distinction between the two types of vectors disappears. As it is, the formal results concerning tensors which are developed are almost trivial to the student of tensor analysis, and at the same time, it seems to me, quite likely to be misleading to the uninitiated in the subject. However, to those interested in the topics of applied mathematics which are treated, and who know the elements of tensor analysis, the work should be of interest.

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