

*Theory of Functionals and of Integral and Integro-differential Equations.* By Vito Volterra. Edited by Luigi Fantappiè. Authorized Translation by Miss M. Long. London and Glasgow, Blackie & Son, Ltd., 1930.

The book at hand is at once a translation into English and a revision and extension of a series of six lectures delivered by Volterra at the University of Madrid in 1925. Its expressed purpose is to develop in the reader an interest in and some familiarity with the theory of functionals. In this the author will in all likelihood be found highly successful.

By way of definition,  $z$  is a functional of the function  $x(t)$ , if under a specifiable law there corresponds one and only one value of  $z$  to each and every function  $x(t)$  defined on a given interval  $a \leq t \leq b$ . The relation

$$z = \int_a^b k(t)x(t)dt,$$

in which the function  $k(t)$  is specified, may be regarded as a simple example.

Beginning with definitions and generalities, the author develops his theme rapidly and carries the discussion over the problems of the functional calculus, the theory of composition and of permutable functions, functionals as a generalization of analytic functions, the theories of integral and integro-differential equations, and a survey of the extensive field of applications.

The presentation retains much of its original character as a series of lectures and is, therefore, a survey, not a treatise. The amount of detail is cut to a minimum, and is adjusted (a mark of notable expository skill) to suffice in each case for the clear presentation of the problem at issue and the method of its resolution, without, however, obscuring the general outlines on which the reasoning is planned.

For the reader versed in the general processes of analysis this book is to be recommended without reservation. An extensive bibliography covering publications to 1929 is included at the end of each lecture.

R. E. LANGER

*Algebra.* By Oskar Perron. Second revised edition. Vol. I, *Die Grundlagen.* Berlin and Leipzig, Walter de Gruyter, 1932. vi+301 pp. 12.80 Mk.

As the first edition of this capable general book on the fundamentals of algebra was reviewed in this Bulletin (vol. 34, p. 115), we shall here speak only of the changes that have been made in this new edition—and of changes that the reviewer wishes had been made. Taken as a whole, the new one is about the same as the old. If anyone familiar with the first edition looks at the revision, he certainly gets the impression that the latter is scarcely more than a reprint with a few secondary editorial changes.

In the third chapter, the author corrects the definition of the product of two determinants of order  $n$  from that in which we use rows of the first with rows of the second to that in which we use rows of the first with columns of the second. He still, however, makes the mistake of defining the product of two matrices (both with  $k$  rows and  $l$  columns) by using rows of the first with rows of the second, although he saves the situation by never applying this definition to