is evanescent as the number of measurements increases. One of the most comprehensive and far-reaching treatments along this line is that of R. von Mises,* whose results, expressible with k-dimensional vectors, are of great generality.

Deltheil does to some extent disentangle his treatment from Bayes' Theorem in subsequent chapters. Error-risk is introduced and *linear* functions of the measurements (p. 101) are considered. It is, indeed, possible to prove rigorously that under the Gaussian Law the arithmetic mean has a probability greater than that of any other linear function or *weighted mean*, in case the "precision" is constant, and to prove the analogous theorem for measurements of unequal precision. So long as we make comparisons only among linear functions, we can move on safe ground.

The subject of least squares, as a practical tool for adjustment of measurements in geodesy and kindred sciences, has a rather well-defined content. Deltheil treats in an admirable manner the topics generally required, giving numerical illustrations in considerable detail. He also sketches a few other topics such as the Gram-Charlier development in Hermite polynomials (pp. 87–90) and Poincaré's method of successive approximations (pp. 94–96). A five-place table of the probability integral concludes the volume.

E. L. DODD

Methoden der Mathematischen Physik. Bd. I, zweite verbesserte Auflage. By R. Courant and D. Hilbert. Berlin, Springer, 1931. xiv+469 pp.

The first edition (1924) of this important and useful book has been already reviewed in a very detailed manner by E. Hille (this Bulletin, vol. 31 (1925), pp. 456–459) and besides is so well known that we may restrict ourselves here to a few remarks. Although the number of pages has not increased considerably (xiii+450 in the first edition) the number of changes is large. Practically all the chapters and sections contain modifications, in exposition as well as in the order of the material. Among the most important additions the following should be mentioned separately: proof of the completeness of the sets of Laguerre's and Hermite's polynomials (Chapter II, 9.6); transformations of problems of the calculus of variations (Chapter IV, 9) where, on the basis of a recent paper by K. Friedrichs (Göttinger Nachrichten, 1929, pp. 13–29) it is shown that in many cases a minimum problem can be transformed into an equivalent maximum problem, with the same value of the extremum in question. The occurrence of a continuous spectrum in problems of mathematical physics is illustrated, of course, by the example of Schrödinger's equation (Chapter V, 12.4). In Chapter VI, 5, a generalized Schrödinger equation is treated from the point of view of the calculus of variations. However, no satisfactory treatment of the continuous spectrum is obtained in this fashion.

The bibliographical references are a little more complete in the present edition than in the first one. In this connection the reference to an unpublished paper by R. G. D. Richardson should be welcomed (p. 404). This paper was

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* *Fundamentalsätze der Wahrscheinlichkeitsrechnung*, Mathematische Zeitschrift, vol. 4 (1919), pp. 1–97.

submitted to the editors of the Mathematische Annalen shortly before the World War. It contained numerous points of contact with the results of Chapter VI (Dependence of the characteristic values of elliptic boundary value problems on the domain and on the coefficients of the problem, zeros of the fundamental functions and the like). A reference to a recent paper by S. A. Janczewsky (Annals of Mathematics, vol. 29 (1928), pp. 521–542, particularly p. 542) would not be out of place on page 393 in connection with the corrected statement of a property of the nodes of the fundamental functions. It was Janczewsky who pointed out that the corresponding statement on page 365 of the first edition was not correct.

Some of the mistakes of the first edition are corrected in the present one, while some others are not. It is a little disappointing, for instance, that the erroneous treatment of zeroes of Bessel's functions (pp. 427–428) and an inconsistent definition of the Legendre's functions of the second kind (p. 436), which were pointed out by the reviewer of the first edition, are preserved in the second edition. A certain elegant vagueness of the style, which was also noticed by the reviewer of the first edition, of course, is preserved in the second one. It is desirable, however, that the statements of some results be made more precise. For instance, the statement of Picard's necessary and sufficient conditions for the existence of a solution of a Fredholm integral equation of the first kind is not correct as it stands on pages 135–136. The boundedness of the integrals $\int K(s, t)^2 ds$, $\int K(s, t)^2 dt$ on page 129 is not needed for the possibility of extension of the classical results of Fredholm theory to unbounded kernels. The statement concerning the sequences converging in mean (p. 93), which was erroneous in the first edition (pp. 96–97), is now correct, but fails to emphasize the essential fact that the function $f(x)$ to which the sequence converges in mean is uniquely determined, and that the whole property is merely a modification of Cauchy's criterion for the existence of a limit of a sequence, and states in essence the completeness of Hilbert's space.

The present reviewer shares with his predecessor the eager expectation (not yet realized) of the promised Volume II of the treatise. A quantity of most important facts concerning the existence of solutions and the precise formulation of the conditions under which these solutions exist are referred to the second volume. Without this volume at hand, the reader can not help the feeling of a certain lack of a solid foundation for many theories developed in the first volume. Hence, thankful as we are for the completion of the present second edition of the first volume of this excellent treatise, we have to agree with one of the authors of the book (R. Courant): "Das Erscheinen dieser zweiten Auflage legt mir mit verdoppelter Stärke die Verpflichtung auf, den zweiten Band, mit dem zusammen dieser vorliegende erst ein abgeschlossenes Ganzes bilden wird, in Druck zu geben."

J. D. TAMARKIN

Algebraische Theorie der Körper. By Ernst Steinitz. Edited by Reinhold Baer and Helmut Hasse. Berlin and Leipzig, de Gruyter, 1930. 150+27 pp.

The present book is a separate edition of Steinitz' well known paper on abstract fields, which appeared in the Journal für Mathematik (vol. 137 (1910)). Steinitz's paper has given rise to numerous investigations on abstract fields and