

Theorie und Anwendung der unendlichen Reihen. By Konrad Knopp. Third edition. Berlin, Springer, 1931. 12+582 pp.

The first edition of this book which appeared in 1922 was reviewed in this Bulletin, (vol. 29 (1923) p. 474). Since then there has appeared also an English translation of the second edition.

The earlier chapters of the new edition are in the main identical with those of the first edition, there being improvements in expressions, additional more recent references, and similar changes. Most of the sections which appeared in the first edition which were not of the "notes and remarks" type have properly been elevated to an equality with the remainder of the text by appearing in the larger print. The main changes have occurred towards the end of the book. The last chapter of the first edition dealing with summability of divergent series was rewritten and rearranged in the second edition, giving a well-ordered, general view of the different definitions of summability and a fairly detailed discussion of the Cesàro and Abel processes. The edition under review has been improved by giving a recent very simple proof of the equivalence of the Cesàro and Hölder summability definitions of the same order due to I. Schur and A. F. Andersen, as well as a recent elegant deduction of the Littlewood Theorem that "if $\sum a_n$ is Abel summable, that is, $\lim_{x \rightarrow 1-0} \sum a_n x^n$ exists, and if the na_n form a bounded sequence, then $\sum a_n$ is convergent in the usual way", due to Karamata.

The last chapter of the new edition, dealing with asymptotic series, was added originally to the English edition, and is here practically unchanged. Beginning with a careful treatment of the Euler sum formula, the chapter leads up to a satisfying introduction to the notion of asymptotic series and their principal properties. With the addition of this chapter the book has increased its usefulness as a carefully executed and well presented introduction to a very fascinating and important branch of analysis.

T. H. HILDEBRANDT