MACROBERT ON HARMONIC FUNCTIONS

Spherical Harmonics. An elementary treatise on Harmonic Functions. By T. M. MacRobert. New York, Dutton, 1928. xii+302 pp. $4.50.

The subject matter of the book falls naturally into three main divisions. The first three chapters deal with Fourier series and applications, Chapters 4 to 13 are on spherical harmonics and the remainder of the book, Chapters 14 to 16, treats of Bessels Functions with applications. As the author states in the preface, he limits himself to those parts of the theory which can be developed expediently without the use of contour integration.

Chapter 1, on the theory of Fourier series, is a very careful and lucid exposition of the elementary theory. The author derives the expansion in Fourier series of the function $f(x)$ subject to the Dirichlet conditions. Here, as throughout the book, he is careful to keep before the reader the validity of the formulas as actually derived in the text. This is a very welcome quality in the book, a quality which is so often lacking in texts designed to appeal to the applied mathematician.

In Chapter 2 the partial differential equation of heat conduction is derived, and the theory of the previous chapter used to obtain solutions for a number of different bodies and with a variety of initial and boundary conditions.

Chapter 3 is a very interesting presentation of the theory of the vibrations of a string. The author solves the wave equation for the case of the harp, the violin, and the piano and brings out strikingly the variations in the results due to the different initial conditions for each of the three instruments.

Chapter 4 begins the study of Spherical Harmonics. Legendre and Laplace coefficients are introduced and the hypergeometric function is studied to some extent. The author departs momentarily from his usually clear style near the bottom of page 79 where the Beta function suddenly appears without explanation.

In Chapter 5 the Legendre Polynomials are studied in considerable detail and in Chapter 6 the Legendre functions of the first and second kind are introduced.

In Chapter 7 the solutions of Legendre's associated equation are investigated quite fully. The Zonal, Tesseral, and Sectorial Harmonics are presented and the expression of a surface spherical harmonic in terms of them obtained.

Chapter 8 begins the application of the theory of the preceding four chapters to mathematical physics. Chapters 8 and 9 contain the proofs of some of the fundamental theorems of potential theory and expressions for the potentials of various bodies, particularly of spheres, spherical shells and spheroids. The second boundary problem for the sphere is also solved in terms of spherical harmonics.

Chapter 10 begins the application of spherical harmonics to electrostatics. The expressions for the surface distributions and potentials for spheres, spherical shells and spheroids are obtained.

In Chapter 11 elliptic coordinates are used to obtain expressions for the potentials and surface distributions of ellipsoids of revolution; and in Chapter 12 the corresponding problem for eccentric spheres is handled by means of bipolar coordinates.

Chapter 13 is devoted to a brief account of Maxwell's theory of spherical harmonics.

In Chapter 14 the expressions for the Bessel functions of the first and second kind are derived and the fundamental properties and formulas involving these are proved.

Chapter 15 is concerned with the modified Bessel functions and asymptotic expansions. The chapter also contains a very lucid account of the zeroes of Bessel functions. Fourier-Bessel expansions are discussed very briefly.

Chapter 16 concludes the book with the application of Bessel functions to the vibration of a circular membrane and to the flow of heat in a circular cylinder and in a sphere.

Very few misprints were found, all of such an obvious nature that it seems useless to list them. It may be worth mentioning, however, that the first line on page 164 should be the last line.

The theory is well illustrated by worked examples in the text and most of the chapter have at the end a good list of exercises. In a number of instances throughout the book the reviewer would have liked to see the general term of an infinite series derived rather than merely the first three or four terms,but nevertheless the author has handled a subject containing mass of detail ln a very clear and skillful manner. The book should appeal strongly to the applied mathematician and to the mathematical physicist. They will find here a scholarly treatment of the type of problems arising in a great many branches of theoretical physics and the tools whereby such problems may be attacked.

The one fault which the reviewer finds with the book is the almost entire lack of references. Except for the mention of four treatises in the preface and a few historical references in the text no explicit reference is made to the literature of which the book treats. The literature is of course so vast that an exhaustive bibliography would be impracticable in a text of this nature. However, the material of the text is capable of a great deal of amplification in so many directions that a few well chosen references would, the reviewer feels, increase its usefulness both to the student of pure mathematics and to the one interested mainly in the applications.

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