

## LIE'S WORKS, VOLUME VI

*Sophus Lie: Gesammelte Abhandlungen.* Edited by Friedrich Engel and Poul Heegaard. Volume VI, edited by Friedrich Engel. Leipzig, Teubner and Oslo, Aschehoug, 1927. xxiv+940 pp.

In this edition of the works of Sophus Lie his published memoirs are to appear in six volumes while a seventh is to be devoted to supplementary matter. Volumes I and II are to contain those of his memoirs in which the geometrical elements predominate, while volumes III and IV are given to differential equations and volumes V and VI to transformation groups. Volumes III and V have previously appeared; and brief reviews of them are to be found in this Bulletin (vol. 29 (1923), pp. 367-369, and vol. 31 (1925), pp. 559-560, respectively). Volume VI is the third to be published, and it is to be followed by volume IV, on which work is already in progress.

In each of the three divisions the arrangement of the material is mainly chronological. The matter in the first of two related volumes is that which was first published at Christiania while the second is composed principally of the memoirs which appeared in *Mathematische Annalen* and the publications of the Leipzig Akademie. In this way one avoids having in a single volume two memoirs one of which is mainly a reworking of the other.

The editor's preface of sixteen pages gives a brief analysis of the contents of the present volume VI. In volumes V and VI together one has all the memoirs of Lie on both finite and infinite continuous transformation groups. In these volumes have also been included many works having to do with the applications of the theory of transformation groups to the theory of differential equations. In these Lie sets forth the point of view, with a development of its consequences, from which he approached the theory of transformation groups and their use for the treatment of differential equations. His remaining memoirs on differential equations are assigned to volumes III and IV.

The memoirs on finite continuous transformation groups have for the most part been worked into Lie's large three-volume treatise on the theory of transformation groups (1888-1893), where a nearly complete development has been given in a systematic treatment. The memoirs on infinite continuous groups have not been incorporated in a systematic development, but here the treatment in the memoirs themselves is sufficiently systematic largely to offset this deficiency.

The editor's carefully prepared analysis (pp. vii-xxii) of the contents of this volume relieves the reviewer of the necessity of such a discussion, particularly since that analysis is more adequate than anything which could be said here in the space which might appropriately be used for such a purpose.

The extreme care with which the editorial work has been done—evidently a labor of love—is again shown by the extent of the notes (pp. 755-940); these clear up many points of difficulty and sometimes give

important matter relating to the development of the theories in Lie's mind. Especially useful in this respect is the account given (pp. 777-793) of the antecedents of memoirs II and III on differential invariants and the differential equations which admit a finite continuous group. This account is mainly in the form of excerpts from letters from Lie to Mayer and Klein. Noteworthy is Lie's insistence (p. 781) on the importance of the synthetic element in the origin of his discoveries. Attention may also be called to the account which Lie gives (pp. 781-782), in a letter to Mayer, of the order in which his memoirs may best be studied.

A free translation of a passage (pp. xxi-xxii) from the editor's introduction will not be without interest:

"If one should go through the whole history of mathematics, I believe that he will not find a second case where, from a few general thoughts, which at first sight do not appear promising, has been developed so extensive and wide-reaching a theory. Considered as an edifice of thought Lie's theory is a work of art which must stir up admiration and astonishment in every mathematician who penetrates it deeply. This work of art appears to me to be a production in every way comparable with that . . . of a Beethoven . . . . It is therefore entirely comprehensible if Lie . . . was embittered that 'deren Wesen, ja Existenz, den Mathematikern fortwährend unbekannt zu sein scheint' (p. 680). This deplorable situation, which Lie himself felt so keenly, exists no longer, at least in Germany. In order to do whatever lies in my power to improve the situation still further, . . . I have sought to clarify all the individual matters (Einzelheiten) and all the brief suggestions in these memoirs."

R. D. CARMICHAEL

#### BORN ON ATOMIC DYNAMICS

*Problems of Atomic Dynamics.* I. *The Structure of the Atom* (20 lectures).  
II. *The Lattice Theory of Rigid Bodies* (10 lectures), by Max Born.

(Delivered at the Massachusetts Institute of Technology in 1925-26, and published by the Institute.) 8vo., 200 pp.

The name of the author of this volume is alone sufficient to insure an authoritative presentation of the subjects which it treats. As a synopsis—followed rather closely—of a course of thirty lectures, it has the faults and the merits which such a method of development involves. Each lecture is, to a certain extent, devoted to some special phase of the subject, and this allows the reader to find out easily what the author has to say on any topic. For anyone who is familiar with the general lines which atomic physics has taken during the last ten years this has decided advantages. On the other hand, it produces some feeling of a lack of connectedness. A mathematician who wishes to learn of the developments which have taken place up to a couple of years back is advised to read first some general descriptive account and then to turn to Professor Born's volume. If he has some previous acquaintance with the mathematical methods of which use is made, he should have a sufficient basis to find out what developments the