BIRKHOFF ON RELATIVITY

The Origin, Nature, and Influence of Relativity. By G. D. Birkhoff. New York, Macmillan, 1925. ix+185 pp.

This elementary introduction to the principles underlying the theory of relativity is based on six lectures delivered at the Lowell Institute in the fall of 1923 and on eight lectures delivered nearly a year later at the Southern Branch of the University of California. In the preface the author says "The degree of general interest manifested, even in the two more technical lectures given at Los Angeles, seemed to indicate the desirability of revising, extending, and unifying this material in book form. My friend Professor Hedrick encouraged me to carry out such a project, and the book now lies before the reader as part of the Macmillan series of mathematical publications of which he is editor."

The book is characterized by its insistence upon the current trend of thought towards a deeper unification of science, by the careful analysis which is made of the way in which one abstraction in science grows out of and includes earlier abstractions, by the clarity and non-technical character of the exposition of the underlying principles of relativity, and by the "formulation of a definite theory of the structure of space and time, of matter and electricity."

The great abstractions which lie back of the theory of relativity and are presupposed in its development are expounded briefly and clearly and in a masterly way. A certain important type of relativity involved in the geometry of Euclid is set forth with precision, with a backward reference to the insistence of Pythagoras upon the fundamental importance of number in the study of natural phenomena. One passes naturally from Euclid to the type of relativity which is embodied in the Newtonian mechanics as it has actually been developed. An analysis is made of the character of the space in which Newton's first law of motion can be valid. Then comes Faraday as the foremost among those who have investigated the facts concerning electricity and magnetism. Since he was lacking in mathematical training "it required his mathematical interpreter Maxwell to give Faraday's concretely expressed 'lines of force' an exact and adequate statement." In the formulation by Maxwell of the laws of electricity and magnetism there is involved a more comprehensive doctrine of relativity of natural law than that of Euclid or that of Newton. From the relativistic elements present in the great doctrines of Euclid, Newton, and Faraday-Maxwell, the author proceeds by steps having the character of inevitableness to the special and then to the general theory of relativity. This process of step-by-step development is so set forth that one can not fail to have a vivid impression of the increasing unity of scientific thought and of the way in which one abstraction is built upon and includes those which precede it. It is not a question of discarding the old for the new but a process of developing and extending the old so that it includes the new. In this is involved implicitly a profound principle of the philosophy of scientific development. These results are achieved in the first chapter and the facts attain an increasing clarity as the exposition proceeds.

The nature of space and time is treated in the second chapter. The third contains an exposition of the old and the new theories of gravitation. These are followed in order by two chapters, one on the experimental tests of relativity and one on some relativistic paradoxes and their explanations. A lucid account and explanation of the difficulties connected with the notion of simultaneity of events is one of the most pleasing features of this analysis of the paradoxes.

These first five chapters are entirely non-technical in their exposition and they put the main ideas of relativity, correctly stated, in a form "not beyond the comprehension of any one possessing a modicum of scientific training."

The essential simplicity of the principles underlying the general theory of relativity is effectively insisted upon. It is not denied that the detailed development of the theory involves complexity. But in this respect the theory of relativity is not alone. This is illustrated by the fact that the determination of the exact mode of vibration of a tuning-fork has never been made. "It is not even possible to explain its general behavior save by elaborate theoretical considerations, nor to specify the fundamental note in advance by any formula."

The sixth and seventh chapters contain the most specific contribution made in the book. In them a definite theory of space and time, of matter and electricity, is formulated. It is practically entirely in these two chapters that mathematical formulas are to be found; but even in these a significant portion of the contents can be understood by one to whom these formulas are meaningless. The author expects to elaborate this theory more fully in a later publication.

The final Chapter VIII is devoted to an analysis of the philosophical influence of relativity. Much which might be profitably said upon this subject is omitted; but several important conclusions are drawn which one is not accustomed to find elsewhere. In view of the growth of abstractions and their inclusion of previous abstractions, illustrated in the most effective way in the development of the theory of relativity, the author concludes: "Relativity does not suggest then that ideals are relative and shifting, but rather that they will enlarge from time to time." He then adds the following:

"The great variety of ways in which one and the same abstraction may be approached indicates that the real differences of underlying opinion at the social level can only be inferred when there exist practical decisions that are different. Undoubtedly this fact should make for sympathy and tolerance. · · · The true role of the systematized abstraction has become more apparent, and freedom has been won to use a formula or a theory and yet not be enslaved to it."

The author is justified in his hope (p. vi) "that the book will prove useful in a mathematical, physical, or philosophical course covering this field, as well as of interest to the general scientific reader." This should be apparent from what has been said in this review. Moreover, I agree heartily with the concluding sentence of the preface: "The field is indeed a most fascinating one, and I believe a course of this character must be given in all our colleges and universities if we are to be fair to our students." The reviewer believes that departments of mathematics, where they have competent instructors, should take the lead in organizing such courses. This book could well furnish a part of the basis for such a course.

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DICKSON ON MODERN ALGEBRA

Modern Algebraic Theories. By L. E. Dickson. Chicago, New York, Boston, Sanborn, 1926. ix+276 pp.

At last English speaking students have a clear, concise guide to the essentials of each of the great branches of modern algebra, written by an authority who has himself enriched several of them by his own outstanding contributions. This publication of Professor Dickson's matured synopsis of modern algebraic theories is a notable event for American mathematics in at least two respects. First, the book will doubtless be for many years the vade mecum of successive generations of graduate students seeking to penetrate the wide and increasingly significant domain of modern algebra. Second, it may suggest to other publishers that it pays to serve mathematics by the publication of something better than inflated advertising matter and mediocre sophomore texts which live their two or three semesters and become liabilities. As it is but seldom that an American publishing house shows sufficient interest in the advancement of science to bring out a work of the calibre of this book, it is to be hoped and expected that the mathematical public will express its appreciation of the publisher's farsighted enterprise in a tangible manner.

Linear algebras, and certain other advanced parts of modern algebra are not discussed in the present treatise; for these the reader is referred to the author's *Algebras and their Arithmetics* (soon to appear in amplified form in a German translation). Presupposing only a knowledge of the rudiments of the calculus and of the elementary theory of equations, *Modern Algebraic Theories* gives rapid, crystal-clear introductions to algebraic invariants, "higher algebra" as usually understood in America, the Galois theory of algebraic equations, the theory of finite linear groups including Klein's theory of the icosahedron and equations of the fifth degree, and group characters.

In the first chapter of only 23 pages the leading concepts of algebraic invariants in the non-symbolic notation are laid down so swiftly that on pp. 17–20 the author obtains the fundamental systems of covariants for the binary p-ic, $p<5$, exhibits the syzygies between them, and on page 23 reaches the solution of the quartic equation from the canonical form of the binary quartic, having defined and proved the covariance of Hessians, Jacobians and discriminants on the way, all with a minimum of computation. Only the essentials are treated; the swarms of insignificant minutiae in which less experienced writers revel, are ignored. The second chapter drives