THE FORTY-SEVENTH REGULAR MEETING OF THE SAN FRANCISCO SECTION

The forty-seventh regular meeting of the San Francisco Section was held at the University of California on Saturday, October 31, 1925. The chairman, Professor E. R. Hedrick, presided at the early part of the meeting, the chairman-elect, Professor Blichfeldt, at the later part. The total attendance was thirty-two, including the following twenty-six members of the Society:

Alderton, Bernstein, Blichfeldt, Buck, Cajori, Corbin, M. W. Haskell, E. R. Hedrick, Hotelling, Frank Irwin, Lehmer, Sophia Levy, Libby, McCarty, E. K. W. McDonald, J. H. McDonald, Moreno, F. R. Morris, Noble, Pehrson, T. M. Putnam, Schmiedel, Pauline Sperry, Stager, A. R. Williams, Wong.

The election of officers for the coming year resulted as follows: Chairman, Professor H. F. Blichfeldt; Secretary, Professor B. A. Bernstein; Program Committee, Professors E. T. Bell, E. R. Hedrick, W. A. Manning, B. A. Bernstein (ex officio).

The resolution adopted April 4, 1925, requiring that the retiring Chairman deliver an address before the Section, was rescinded, and a motion was passed empowering the Program Committee to invite a member to deliver a paper at length at the October meeting of the Section.

The Spring meeting in the Northwest will be held at the University of Washington on June 12. It was decided to hold the next Fall meeting at the University of California on October 30, 1926.

Titles and abstracts of papers read at the meeting follow.

1. Dr. Harold Hotelling: *Multiple-sheeted spaces and manifolds of states of motion.*

Let two sheets of a 3-space covering a space of inversion join along a twisted strip. If a tore T in one sheet be deformed through a non-singular set of positions of which the strip, counted twice, is one, into a tore T' in

the second sheet covering T, then the transformation of T into T' is determined topologically by the amount of twist in the strip. Using such transformations the author shows how to find covering spaces which are homeomorphic with manifolds of genus one. As a second application, he finds simplified covering spaces which represent manifolds of states of motion.

2. Dr. Harold Hotelling: *Solution of a problem proposed by Keynes.*

If the probability of a successful trial be taken as the number of past successes divided by the number of past trials, then the probability after a successes and b failures that in $p+q$ further trials there will be p successes and q failures is not, as sometimes stated, given by Bernoulli's theorem, but by a certain function $\phi(p, q)$, where $\phi(p, -1)=\phi(-1, p)=0$. The problem is raised by Keynes (*A Treatise on Probability*, 1921, p. 351). The author finds the solution and the exact form of $\phi(p, q)$.

3. Dr. Harold Hotelling: *Two generalizations of the Pearsonian correlation coefficient.*

The correlation coefficient may be written

$$r=g_{ij}\delta x^i\delta y^j/\sqrt{g_{pq}\delta x^p\delta x^q\cdot g^{st}\delta y^s\delta y^t},$$

where $g_{ij}=0$ if $i\neq j$, $g_{ii}=1$, summation is denoted by the repetition of an index, and δx^i and δy^i are the deviations of the observations x^i and y^i from their respective means. The author gives two generalizations of this coefficient: one generalization is to allow g_{ij} to take arbitrary values; the other is to remove the restriction on δx^i and δy^i. The first is a part of the generalization of the theory of errors and least squares which arises when the sum of squares is replaced by a general quadratic form. This is applicable when (as is usually the case) the observations are not strictly independent in the probability sense. The question of validity of certain very important statistical methods depends upon determining the distribution of this generalized r in a certain curved hyperspace. The second generalization is applicable for example in biometrics, where it is more rational to take δx^i as a deviation of a character from the mean value for a whole population, if this is known, than from the mean of the sample.

4. Dr. Harold Hotelling: *Mixed multiple-partial correlation.*

The author shows that by the use of tensor notation the theory of multiple correlation may be presented very compactly. He finds by this method an expression believed to be new for the multiple correlation which a variate z has with the variates $x_1, \cdots, x_m$ after elimination of the effect of p other variates $y_1, \cdots, y_p$.

5. Professor Florian Cajori: *A notable case of finger-reckoning in America.*

The author gives an account of a booklet by a Mexican priest, Buenaventura Francisco de Ossorio, entitled *Astronomica, y Harmoniosa Mano*

(Mexico, 1757), in which a novel and very specialized use is made of the left hand in determining the dates of movable feasts, for any year in the Gregorian calendar.

6. Professor J. H. McDonald: *The reduction of abelian integrals of genus* 2.

If an abelian integral is given it is not possible, in general, to determine whether it is reducible or not. If the integral is of genus 2 and reducible it is known that the sextic must satisfy one of an infinite number of conditions which are algebraic. The decision as to reducibility must depend on the formation of these conditions since it is not possible to determine the linear relations between the periods, which exist in the case of reduction. The algebraic conditions referred to seem to have been found only for the cases where the transformation effecting the reduction is of order 2, 3, or 4. They are here found for a transformation of any order by a method which consists in constructing the reducing substitution by evaluation of certain integrals of the third kind. If the reduction is possible recurrence must exist between the results found from which the required condition for reducibility is deduced for any particular order by a uniform process. The form of the transformation reducing the second reducible integral, which is known to exist, is also found at the same time.

7. Professor D. N. Lehmer: *On the construction of factor stencils.*

This paper will appear in full in the present issue of this BULLETIN.

8. Professor H. F. Blichfeldt: *The minimum value of positive quadratic forms in seven variables.*

In the present paper the author concludes his investigations on the minimum values of positive quadratic forms in n variables (cf. this BULLETIN, vol. 31 (1925), p. 386). The absolute minimum of a quadratic form of determinant D in 7 variables, for integral values of the variables, is found to be $\sqrt[7]{64\,|D|}$. Approximate minima of forms for $n>7$ are deduced.

9. Professor T. M. Putnam: *A proof of the second mean-value theorem.*

The author shows that the second mean-value theorem is obtainable from the first mean-value theorem by integration by parts. The theorem is proved for the case where one function is monotonic in the interval and the other possesses a derivative. The second mean-value theorem thus appears in this case as a direct corollary of the first.

10. Professor M. W. Haskell: *Felix Klein. An appreciation.*

This paper will appear in full in an early issue of this BULLETIN.

A. B. BERNSTEIN,
Secretary of the Section.